TA Session 8

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ECON 8040: Macroeconomics I
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Today's Session

- **Problem Set 2** grades on eLC
- **Problem Set 3** grades on eLC
- **Problem Set 4** due Friday, Oct. 10 at 11:59p.m.
- **Midterm Exam** on Thursday, Oct. 16, 2:20–3:35p.m. in MLC 275

Problem 1: Solow Closed Economy

(a) Find expression for capital per worker

- Given
  - Law of motion: $K_{t+1} = (1-\delta)K_t + I_t$
  - Investment: $I_t = sY_t$
  - Production function: $Y_t = K_t^{\alpha} N_t^{1-\alpha}$
  - Population: $N_{t+1} = (1+n)N_t$
- Replace invesment and production function in law of motion

$$K_{t+1} = (1-\delta)K_t + sK_t^{\alpha} N_t^{1-\alpha}$$

- Divide both sides by N_{t+1} to arrive to solution

$$k_{t+1} = \frac{(1-\delta)k_t + sk_t^{\alpha}}{1+n}$$

Problem 1: Solow Closed Economy (cont.)

(b) Find steady state capital / output per worker

- Impose steady state condition ($k_t = k_{t+1}$) on equation from (a)

$$k^* = \frac{(1-\delta)k^* + s\,(k^*)\,\alpha}{1+n}$$

$$\implies k^* = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$\implies y^* = \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Problem 1: Solow Closed Economy (cont.)

(c) Plot evolution of aggregate output Y_t over time

- At what rate does **aggregate output** grow?
  Use intuition or show w/ math:

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}^{\alpha} N_{t+1}^{1-\alpha}}{K_t^{\alpha} N_t^{1-\alpha}} = \frac{k_{t+1}^{\alpha} N_{t+1}}{k_t^{\alpha} N_t} = 1 + n \text{ in steady state}$$

$$\implies Y_t = (1+n)^t Y_0$$

Problem 1: Solow Closed Economy (cont.)

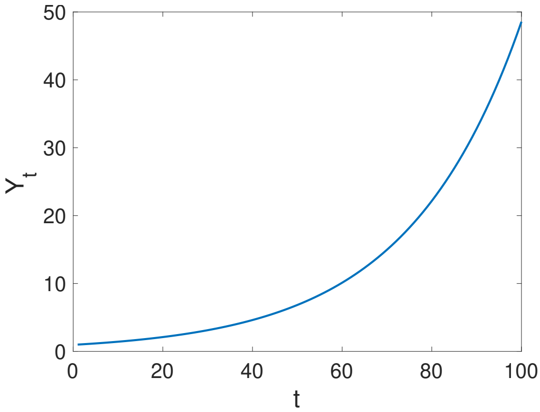

Figure: Evolution of Aggregate Output

Problem 1: Solow Closed Economy (cont.)

(d) Compare 2 economies w/ same α, δ, ***s***, and ***n***, but one has 10x pop.

- Steady state capital / output per worker are **equal**
- Aggregate variables are 10x larger in more populous country

Problem 2: Prove Properties of Solow Model

- For (a)–(d), use derivatives of $h(k)$ and assumed properties of $f(k)$

$$
\begin{aligned}
h(k) &= (1-\delta)k + sf(k) \\
h'(k) &= 1 - \delta + sf'(k) \\
h''(k) &= sf''(k)
\end{aligned}
$$

(a) Show $h'(k) > 0$ on $(0, \bar{k}]$

(b) Evaluate $\lim_{k \to 0} h'(k)$

(c) Evaluate $\lim_{k \to \infty} h'(k)$

(d) Show $h''(k) < 0$

Problem 2: Prove Properties of Solow Model (cont.)

(e) Steady state capital k^* satisfies $h(k^*) = k^*$

- Notice $h(k^*) = k^* \Leftrightarrow sf(k^*) = \delta k^*$
- For small $k > 0$, $sf(k) > \delta k$ b/c $sf'(k) \to +\infty > \delta$
- Concave $sf(k)$ guarantees intersection w/ δk for some larger k^*
- B/c δk increasing + linear, $sf(k)$ increasing + concave, k^* unique

(f) Show $\exists\, \bar{k}$ s.t. for $k > \bar{k}$, $h(k) < k$

- Sufficient to show that k^* is one such $\bar{k}$
- $sf(k)$ is concave and $sf'(k^*) < \delta$, so $k > k^* \Rightarrow sf(k) < \delta k \Rightarrow h(k) < k$

(g) There are three cases to show

- By (e), economy has converged if $k_0 = k^*$
- By (f), if $k_0 > k^*$, then $k^* < \cdots < h(h(k_0)) < h(k_0) < k_0$
- Also show that if $k_0 \in (0, k^*)$, then $k_0 < h(k_0) < h(h(k_0)) < \cdots < k^*$

Problem 2: Prove Properties of Solow Model (cont.)

- **Example: Problem 2(c)** Let $h(k) = (1-\delta)k + sf(k)$, where $s, \delta \in (0,1)$ and $f(k)$ is strictly increasing, concave, and satisfies Inada conditions. Show that the slope of $h(k) < 1$ as k gets large.
- Write a good proof with math *and words*:
  1. (Re)state what you want to show

Problem 2: Prove Properties of Solow Model (cont.)

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- Write a good proof with math *and words*:
  1. (Re)state what you want to show
     We want to show that $\lim_{k\to\infty} h'(k) < 1$.
  2. Write down what you are allowed to assume

Problem 2: Prove Properties of Solow Model (cont.)

- **Example: Problem 2(c)** Let $h(k) = (1 - \delta)k + sf(k)$, where $s, \delta \in (0, 1)$ and $f(k)$ is strictly increasing, concave, and satisfies Inada conditions. Show that the slope of $h(k) < 1$ as k gets large.
- Write a good proof with math *and words*:
  1. (Re)state what you want to show
     We want to show that $\lim_{k \to \infty} h'(k) < 1$.
  2. Write down what you are allowed to assume
     By Inada conditions, $\lim_{k \to \infty} f'(k) = 0$, *and we assume* $\delta \in (0, 1)$.
  3. Write steps that *follow from your assumptions* and move towards the statement in (1)

Problem 2: Prove Properties of Solow Model (cont.)

- **Example: Problem 2(c)** Let $h(k) = (1-\delta)k + sf(k)$, where $s, \delta \in (0,1)$ and $f(k)$ is strictly increasing, concave, and satisfies Inada conditions. Show that the slope of $h(k) < 1$ as k gets large.
- Write a good proof with math *and words*:
  1. (Re)state what you want to show
     We want to show that $\lim_{k\to\infty} h'(k) < 1$.
  2. Write down what you are allowed to assume
     By Inada conditions, $\lim_{k\to\infty} f'(k) = 0$, *and we assume* $\delta \in (0,1)$.
  3. Write steps that *follow from your assumptions* and move towards the statement in (1)
     The definition of $h(k)$ *implies* $h'(k) = 1 - \delta + sf'(k)$.
     Therefore, $\lim_{k\to\infty} h'(k) = 1 - \delta + s \lim_{k\to\infty} f'(k) = 1 - \delta < 1$. ■

Problem 2: Prove Properties of Solow Model (cont.)

- Proofs are structured "linearly" (i.e., each statement must follow from what was stated before)
- When creating proofs, you often write (1), then (3), then (2)
- I call this writing the proof "backwards"
- In previous example,
  - First, write *We want to show* $\lim_{k\to\infty} h'(k) < 1$
  - Second, replace $h'(k)$ and see that you need to show

$$1 - \delta + s \lim_{k\to\infty} f'(k) < 1$$

  - To reach this conclusion, we need to know (or assume) something about δ and $\lim_{k\to\infty} f'(k)$ $\leftarrow$ **This goes into (2)!**

Problem 2: Prove Properties of Solow Model (cont.)

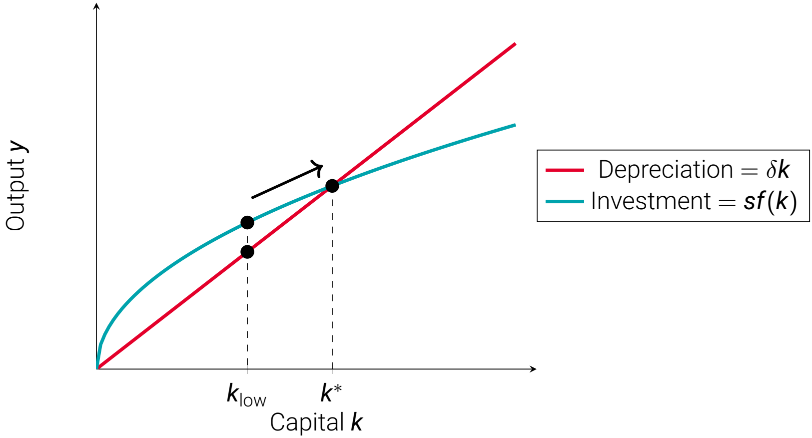

Figure: Converging to Steady State in the Solow Model

Problem 3: Cross-Country Income Differences

(b) Plot 2016 capital-output ratio against ave. savings rate

– Savings rate has positive correlation w/ K/Y as predicted by model

Problem 3: Cross-Country Income Differences (cont.)

(b) Find equation for long-run capital-output ratio

$$\frac{K_t}{Y_t} = \frac{K_t}{ZK_t^{\alpha}(A_tN_t)^{1-\alpha}}$$
$$= \frac{1}{Z} \times \left(\frac{K_t}{A_tN_t}\right)^{1-\alpha}$$
$$= \frac{1}{Z} \times k_t^{1-\alpha}$$

- In long-run, $k_t = k^*$ from (c). Replace and simplify to finish

Problem 3: Cross-Country Income Differences (cont.)

(c) Find steady state capital per effective worker k^*, output per effective worker y^*

- "Detrend" law of motion of capital to be written in terms of variables *per effective worker*
- Impose $k_{t+1} = k_t$ in steady state to find k^*
- Notice $y_t = \dfrac{Y_t}{A_t N_t} = Z(k_t)^\alpha$

Problem 3: Cross-Country Income Differences (cont.)

(d) Assume $\alpha = 1/3$. By (c)

$$Z = \frac{y^*}{(k^*)^{1/3}}$$

Table: Ave. TFP among poorest and richest countries

	Poorest 20%	Richest 20%
Ave. Z	235.2	1,371.6

– Poorest 20% have ≈ **17%** productivity of richest 20%

Problem 3: Cross-Country Income Differences (cont.)

(e) What fraction of income differences are explained by productivity and savings?

- Using equation for y^* from (c), can decompose differences in output per capita into
  - differences in $Z^{\frac{1}{1-\alpha}}$
  - differences in $s^{\frac{\alpha}{1-\alpha}}$
  - interaction between having different productivity and savings
- Differences in productivity explain much more of the gap ($\approx$ 85%) than differences in savings rates ($\approx$ 5%)

Problem 4: Savings Rates & Income Differences

- Assume no pop. growth ($n_t = 1 \; \forall \; t$) and no productivity growth

$$y_t = k_t^{\alpha}$$

(a) Find an equation for $y(s)/y(s = 0.2)$

- Find steady state capital and output

$$k^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}} \qquad y^*(s) = \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

- Simplify $y(s)/y(s = 0.2)$

$$\frac{y^*(s)}{y^*(s = 0.2)} = \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\delta}{0.2}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{s}{0.2}\right)^{\frac{\alpha}{1-\alpha}}$$

Problem 4: Savings Rates & Income Differences (cont.)

(b) Set $y(s)/y(s = 0.2) = 1/30$ and solve s in terms of α

$$\left(\frac{s}{0.2}\right)^{\frac{\alpha}{1-\alpha}} = \frac{1}{30} \Rightarrow s(\alpha) = 0.2 \times \left(\frac{1}{30}\right)^{\frac{1-\alpha}{\alpha}}$$

(c) Plug $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ into $s(\alpha)$

α	0.1	0.3	0.5	0.7	0.9
$s(\alpha)$	0.0000	0.0001	0.0067	0.0466	0.1371

- For savings to explain income gap between U.S. and poor countries, for plausible $\alpha \approx 1/3$, poor countries must have savings rates ≈ 0

Problem 5: Convergence Speed

- Analyze how TFP and α affect transition paths of the economy
- Normalizing $n_t = 1 \, \forall \, t$, $y_t = Ak_t^{\alpha}$
- Steady state capital

$$k^*(\alpha, A; s = 0.2, \delta = 0.08) = \left(\frac{0.2 \times A}{0.08}\right)^{\frac{1}{1-\alpha}}$$

Problem 5: Convergence Speed (cont.)

(a) For each $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ (and fixing $A = 1$),

1. Calculate $k^*(\alpha, A = 1; s = 0.2, \delta = 0.08)$
2. Start at $k_0 = 0.1 \times k^*$
3. Calculate $k_1 = (1 - 0.08) \times k_0 + 0.2 \times k_0^\alpha$
4. Iterate forward until $k_{T-1} < 0.6k^* \leq k_T$
5. Report T

- Do same for other transition paths

Problem 5: Convergence Speed (cont.)

Table: Part (a) Transition Durations

	α				
Transition	0.1	0.3	0.5	0.7	0.9
$0.1k^*-0.6k^*$	13	19	29	54	179
$0.6k^*-0.8k^*$	11	14	20	34	103
$0.8k^*-0.9k^*$	11	14	19	32	94
$0.9k^*-0.95k^*$	11	14	19	31	91
$0.95k^*-0.975k^*$	11	14	19	30	89

Problem 5: Convergence Speed (cont.)

(b) Repeat (a) for $A \in 2, 4$ (and fixing $\alpha = 0.5$)

Table: Part (b) Transition Durations

Transition	A = 1	A = 2	A = 4
$0.1k^* - 0.6k^*$	29	29	29
$0.6k^* - 0.8k^*$	20	20	20
$0.8k^* - 0.9k^*$	19	19	19
$0.9k^* - 0.95k^*$	19	19	19
$0.95k^* - 0.975k^*$	19	19	19

Problem 5: Convergence Speed (cont.)

(c) Why different effects? A doesn't affect convergence speed, α does

- Growth of capital $g(k) \equiv \frac{h(k)-k}{k} = sAk^{\alpha-1} - \delta$
- First-order Taylor expansion around steady state k^*:

$$\begin{aligned} g(k) &= \underbrace{g(k^*)}_{=0} + g'(k^*)(k-k^*) \\ &= \left(\frac{(\alpha-1)sA(k^*)^{\alpha-1}}{k^*}\right)(k-k^*) \\ &= (\alpha-1)sA\left(\frac{\delta}{sA}\right)\left(\frac{k-k^*}{k^*}\right) \\ &= (\alpha-1)\delta\left(\frac{k-k^*}{k^*}\right) \end{aligned}$$

- Thus $\frac{\partial g(k)}{\partial \alpha} = \delta\left(\frac{k-k^*}{k^*}\right) < 0$ for $k < k^*$ and $\frac{\partial g(k)}{\partial A} = 0$

Problem 5: Convergence Speed (cont.)

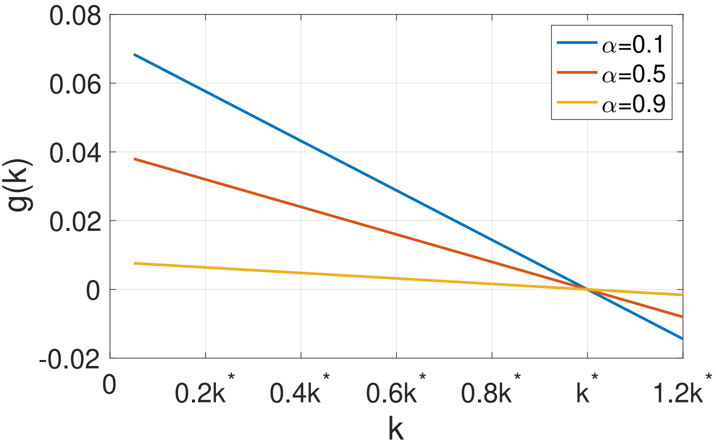

Figure: Convergence speed is decreasing in α

Problem 6: Solow Economy w/ Affine Production

(a) Derive and plot $h(k)$ when $f(k) = Ak$

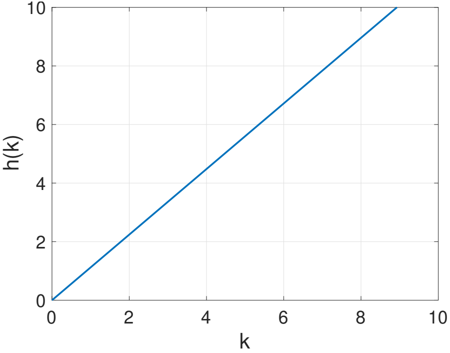

Figure: $h(k) = (1 - \delta + sA)k$ for $s = 0.2$, $\delta = 0.08$, $A = 1$

Problem 6: Solow Economy w/ Affine Production (cont.)

(b) Argue economy does not have positive steady state

- Steady state exists iff $h(k) = k \Leftrightarrow \delta = sA$
- If $\delta = sA$, economy's capital forever equals endowment k_0
- If $\delta \neq sA$, then steady state does not exist
- In either case, economy does not "converge" toward steady state

(c) Show that if $sA > \delta$, then capital is unbounded

- Capital growth is $\frac{h(k)-k}{k} = sA - \delta > 0$
- Thus capital never stops growing
- **Growth is balanced** b/c output growth = capital growth

$$\frac{y(h(k)) - y(k)}{y(k)} = \frac{A(1 + sA - \delta)k - Ak}{Ak} = \frac{A(sA - \delta)k}{Ak} = sA - \delta$$

(d) What happens if $sA < \delta$? **Capital decumulates**

Problem Set 3

- Two **dynamic optimization** problems
  1. Finite-horizon planner problem
  2. **Finite-horizon** consumption-saving problem
- Both “dynamic” due to **capital/asset accumulation**
  - Today’s savings determine resources for future consumption
  - **Euler equation** characterizes optimal intertemporal balance

Problem 1: Finite-horizon planning problem

(a) Solve the optimal savings path
- Write FOC wrt planner's choice k_{t+1}
  - Notice that k_{t+1} appears twice in objective

$$\max_{\{k_{t+1}\}_{t=0}^{T}} \log(Ak_0^\alpha - k_1) + \cdots + \beta^t \log(Ak_t^\alpha - k_{t+1}) + \beta^{t+1} \log(A{k_{t+1}}^\alpha - k_{t+2}) + \ldots$$

- Write equation w/ $z_{t+1} \equiv \frac{k_{t+2}}{Ak_{t+1}^\alpha}$ on left side and z_t on right side
- Write z_t in terms of α, β, and z_{t+1}

$$z_t = \frac{\alpha\beta}{1 + \alpha\beta - z_{t+1}}$$

Problem 1: Finite-horizon planning problem (cont.)

- Solve z_t in terms of α, β, t, and T by **backwards induction**
  - Using $z_T = 0$ (**why is this true?**), solve z_{T-1}

$$z_{T-1} = \frac{\alpha\beta}{1+\alpha\beta}$$

  - Use z_{T-1} to find z_{T-2}, and so on

$$z_{T-2} = \frac{\alpha\beta}{1+\alpha\beta-\left(\frac{\alpha\beta}{1+\alpha\beta}\right)} = \frac{\alpha\beta(1+\alpha\beta)}{1+\alpha\beta+(\alpha\beta)^2}$$

$$z_{T-3} = \frac{\alpha\beta(1+\alpha\beta+(\alpha\beta)^2)}{1+\alpha\beta+(\alpha\beta)^2+(\alpha\beta)^3}$$

  - Recognize pattern to write z_t

$$z_t = \frac{\alpha\beta(1+\alpha\beta+\cdots+(\alpha\beta)^{T-t-1})}{1+\alpha\beta+\cdots+(\alpha\beta)^{T-t}}$$

Problem 1: Finite-horizon planning problem (cont.)

- Evaluate finite geometric series to simplify

$$
\begin{aligned}
z_t &= \frac{\alpha\beta \times \sum_{j=0}^{T-t-1} (\alpha\beta)^j}{\sum_{j=0}^{T-t} (\alpha\beta)^j} \\
&= \alpha\beta \times \frac{1-(\alpha\beta)^{T-t}}{1-\alpha\beta} \times \frac{1-\alpha\beta}{1-(\alpha\beta)^{T-t+1}} \\
&= \frac{\alpha\beta(1-(\alpha\beta)^{T-t})}{1-(\alpha\beta)^{T-t+1}}
\end{aligned}
$$

Problem 1: Finite-horizon planning problem (cont.)

(b) Evaluate and discuss $\lim_{T\to\infty} z_0$, $\lim_{T\to\infty} z_1$

- Notice $0 < \alpha\beta < 1$, so $(\alpha\beta)^T \to 0$ as $T \to \infty$
- For z_0

$$\lim_{T\to\infty} \frac{\alpha\beta(1-(\alpha\beta)^T)}{1-(\alpha\beta)^{T+1}} = \alpha\beta$$

- For z_1

$$\lim_{T\to\infty} \frac{\alpha\beta(1-(\alpha\beta)^{T-1})}{1-(\alpha\beta)^T} = \alpha\beta$$

- Notice this implies $\lim_{T\to\infty} k_{t+1} = \alpha\beta \underbrace{Ak_t^\alpha}_{\text{output}}$
- In the long-run, a constant share of output ($\alpha\beta$) is saved

Problem 2: Consumption-savings problem

(a) Derive the Euler equation w/ FOCs wrt c_t, c_{t+1}, and a_{t+1}

$$c_t^{-\sigma} = \beta(1+r)c_{t+1}^{-\sigma}$$

(b) Show **intertemporal elasticity of substitution** (IES) equals $1/\sigma$

- Use Euler equation to write $\log(c_{t+1}/c_t)$
- Evaluate $\partial \log\left(c_{t+1}/c_t\right)/\partial \log(1+r)$
- **What does the IES represent?** IES determines substitution between present and future consumption when return ratechanges.
  - $\uparrow$ IES $\Rightarrow$ Consumers more responsive to interest rate changes
  - $\downarrow$ IES $\Rightarrow$ Consumers less responsive to interest rate changes

Problem 2: Consumption-savings problem (cont.)

(c) Find optimal consumption in terms of intial assets a_0 and endowments e_t

- Derive **lifetime budget constraint**
  - Combine period budget constraints through savings decisions
    Note: Standard to assume zero return on initial assets a_0

$$c_0 + a_1 = a_0 + e_0$$
$$c_1 + a_2 = a_1(1+r) + e_1$$
$$c_1 + a_3 = a_2(1+r) + e_2$$

  - Write equation relating consumption and a_{T+1} to endowments and a_0

$$\sum_{t=0}^{T} \frac{c_t}{(1+r)^t} + \underbrace{\frac{a_{T+1}}{(1+r)^T}}_{=0} = a_0 + \sum_{t=0}^{T} \frac{e_t}{(1+r)^t}$$

Problem 2: Consumption-savings problem (cont.)

- Find c_0
  - Redefine consumer problem as preference max. s.t. lifetime b.c.
  - Find Euler using FOCs wrt c_t, c_{t+1}

$$c_t = [\beta(1+r)]^{\frac{-1}{\sigma}} c_{t+1}$$

  - Start at $t = 0$ and iterate forward to write c_t in terms of $(c_0, \beta, r, \sigma, t)$

$$\begin{aligned} c_0 &= [\beta(1+r)]^{\frac{-1}{\sigma}} c_1 \\ &= \left([\beta(1+r)]^{\frac{-1}{\sigma}}\right)^2 c_2 \\ &= \dots \\ &= \left([\beta(1+r)]^{\frac{-1}{\sigma}}\right)^t c_t \\ c_t &= [\beta(1+r)]^{\frac{t}{\sigma}} c_0 \end{aligned}$$

Problem 2: Consumption-savings problem (cont.)

- Replace c_t in lifetime budget constraint to solve c_0
  (You need to evaluate finite geometric series)

$$c_0 \times \sum_{t=0}^{T} \frac{[\beta(1+r)]^{\frac{t}{\sigma}}}{(1+r)^t} = a_0 + \sum_{t=0}^{T} \frac{e_t}{(1+r)^t}$$

$$c_0 = \underbrace{\frac{1-\tilde{\beta}}{1-\tilde{\beta}^{T+1}} \times \underbrace{\left(a_0 + \sum_{t=0}^{T} \frac{e_t}{(1+r)^t}\right)}_{\textbf{P.V. Lifetime Wealth}}}_{\textbf{Permanent Income}}$$

$$\text{where } \tilde{\beta} = \beta^{1/\sigma} \times \left(\frac{1}{1+r}\right)^{\frac{\sigma-1}{\sigma}}$$

- Household consumes fraction of its lifetime wealth each period
- Whether consumption grows depends on $\beta \lesseqgtr \frac{1}{1+r}$

Problem Set 4

- Due **Friday, October 10 at 11:59 p.m.**
- Four dynamic programming problems
- All use **guess-and-verify** to solve model analytically
- Finish Problem 1, then do other problems
- If stuck, review lecture notes for case w/ log preference
- Useful log properties

$$\log(a \times b) = \log(a) + \log(b) \qquad \log(k^{\alpha}) = \alpha \times \log(k)$$

$$\log(a/b) = \log(a) - \log(b)$$

Problem 1

(a) Write the Bellman equation

(b) Find value function $v(k)$ and policy function k' by **guess-and-verify**

1. Assume $v(k) = A\frac{k^{1-\sigma}}{1-\sigma}$, find k' using optimality condition
   - It should depend on A, k, and parameters
   - **Hint:** May help simplify math to replace $R \equiv z + 1 - \delta$
2. Replace k' in Bellman and solve for coefficient A
   - A should depend on parameters only, **not** k
3. Replace A in expression from first step to write policy function $g(k)$

(c) Use the policy function for $k' = g(k)$ to find $\frac{k_{t+1}}{k_t}$ and $\frac{c_{t+1}}{c_t}$

Problem 2

(a) Write two things
  - Optimality condition for labor supply n using $F(k, k')$
    - You can't write closed-form solution for n, yet
  - Bellman equation using $F(k, k')$

(b) Assuming full depreciation and $V(k) = A + B\log(k)$, find k'
  - It depends on parameters, B, and n

(c) Write n in terms of parameters and B using results from (a) and (b)

(d) Replace k' and n in guess of $V(k)$ to solve B

(e) Solve for policy functions n, k', and c as function of state k
  - Check your work: $\phi = 1 \Rightarrow n = 1$
  - Check your work: $n \in [0, 1]$ always (assume $\phi \in [0, 1]$, $\alpha, \beta \in (0, 1)$)

Problem 3

(a) Write planning problem $w(k_0, h_0)$
   - k_0, h_0 is given initial stocks of physical, human capital

(b) Write the planning problem recursively

(c) Assume full depreciation ($\delta = 1$) and use guess-and-verify to solve:
   - $V(k, h)$
   - $k'(k, h)$
   - $h'(k, h)$

Problem 4

(a) Rewrite the problem so $\{k_{t+1}\}_{t=0}^{\infty}$ is only choice variable

(b) Write the problem recursively using two equations
  - $v(k, \theta_L)$
  - $v(k, \theta_H)$
  - You know how state θ_t evolves over time

(c) Solve the Bellman equations using guess-and-verify

(d) Find policy functions $g(k, \theta_L)$ and $g(k, \theta_H)$

(e) Assume $k_0 = 1$ and simulate k_1, k_2, k_3
  - Preference scalar θ_t switches deterministically
  - You know $\theta_0 = \theta_H$, $\theta_1 = \theta_L$, so on