TA Session 7

Michael Kotrous
John Munro Godfrey, Sr. Department of Economics
University of Georgia

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Today's Session

- **Problem Set 2** grades to be posted soon
- **Problem Set 3** due Friday, Sept. 26 (tonight) at 11:59 p.m.

Problem Set 3

- Due **Friday, Sept. 26 at 11:59 p.m.**
- Two **dynamic optimization** problems
  1. Finite-horizon planner problem
  2. Finite-horizon consumption-saving problem
- Both “dynamic” due to **capital/asset accumulation**
  - Today’s savings determine resources for future consumption
  - **Euler equation** characterizes optimal intertemporal balance

Problem 1: Finite-horizon planning problem

(a) Solve the optimal savings path

- Write FOC wrt planner's choice k_{t+1}
- Write equation w/ $z_{t+1} \equiv \frac{k_{t+2}}{Ak_{t+1}^{\alpha}}$ on left side and z_t on right side
- Write z_t in terms of α, β, and z_{t+1}
- Solve z_t in terms of α, β, t, and T by **backwards induction**
  - Using $z_T = 0$ (**why is this true?**), solve z_{T-1}
  - Use z_{T-1} to find z_{T-2}, and so on
  - Recognize pattern to write z_t
- Write $k_{t+1} = z_t \times Ak_t^{\alpha}$

(b) Evaluate and discuss $\lim_{T\to\infty} z_0$, $\lim_{T\to\infty} z_1$

- What is happening to share of output that is saved in the long-run?

Problem 2: Consumption-savings problem

(a) Derive the Euler equation
- Combine FOCs wrt c_t, c_{t+1}, and a_{t+1}

(b) Show **intertemporal elasticity of substitution** (IES) equals $1/\sigma$
- Use Euler equation to write $\log(c_{t+1}/c_t)$
- Evaluate $\partial \log\left(c_{t+1}/c_t\right)/\partial \log(1+r)$
- What does the IES represent?

Problem 2: Consumption-savings problem (cont.)

(c) Find optimal consumption in terms of intial assets a_0 and endowments e_t
   - Derive **lifetime budget constraint**
     - Combine period budget constraints through savings decisions
     - Find equation relating P.V. consumption, a_{T+1} to P.V. of endowments, a_0
     - Note that $a_{T+1} = 0$ (for the same reason $z_T = 0$ in #1)
   - Find c_0
     - Redefine consumer problem as preference max. s.t. lifetime b.c.
     - Find Euler using FOCs wrt c_t, c_{t+1}
     - Use Euler to write c_t in terms of c_0, β, r, and σ
     - Replace c_t in lifetime budget constraint to solve c_0 (You need to evaluate finite geometric series)
   - You already wrote c_t in terms of c_0, so you're finished!