TA Session 6

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Today's Session

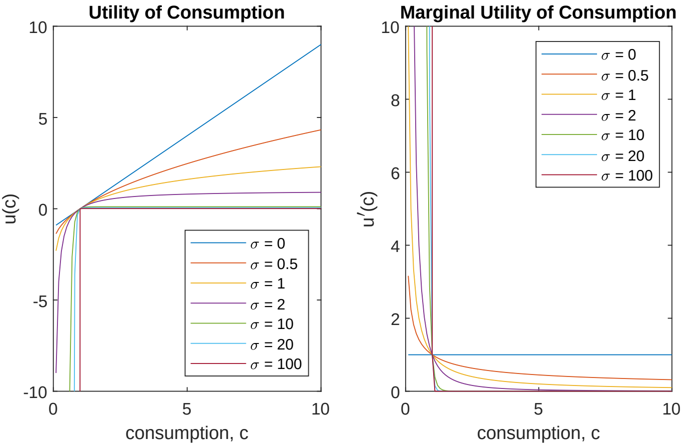
- **Computation Exercise 1** grades on eLC
- **Problem Set 2** due Friday, Sept. 19 (tonight) at 11:59 p.m.
- **Problem Set 3** due Friday, Sept. 26 at 11:59 p.m.

Computation Exercise 1

1. Plot CRRA utility for different relative risk aversion parameters σ
   - $\sigma \geq 0$ is coefficient of relative risk aversion
   - Satisfies **Inada conditions**
   - **Homothetic:** income does not affect ratio of goods consumed
     - Useful for aggregation (see MWG Proposition 4.C.2)
   - Distinct class of utility functions consistent w/ **balanced growth path**
2. Model of ag, manufacturing, and services expenditure shares
   - Parameters $\bar{c}_a$, $\bar{c}_s$ shift marginal utility of consumption
   - Calibrate parameters to match data
3. Newton's Method
   - Numerical method for solving systems of equations

Problem 1

- Plot CRRA utility (and marginal utility)



Problem 2

(a) Find expenditure shares by solving preference maximization

- Write three first-order conditions
- Rewrite budget constraint in terms of one unknown and solve
- Use first-order conditions to finish

$$\frac{p_a c_a}{C} = \frac{\omega_a(C + p_m\bar{c}_m + p_s\bar{c}_s) - (\omega_m + \omega_s)p_a\bar{c}_a}{C(\omega_m + \omega_a + \omega_s)}$$

$$\frac{p_m c_m}{C} = \frac{\omega_m(C + p_a\bar{c}_a + p_s\bar{c}_s) - (\omega_a + \omega_s)p_m\bar{c}_m}{C(\omega_m + \omega_a + \omega_s)}$$

$$\frac{p_s c_s}{C} = \frac{\omega_s(C + p_a\bar{c}_a + p_m\bar{c}_m) - (\omega_a + \omega_m)p_s\bar{c}_s}{C(\omega_m + \omega_a + \omega_s)}$$

Problem 2 (cont.)

(b) Assuming $\bar{c}_a < 0$, $\bar{c}_s > 0$, and $\bar{c}_m = 0$, how do shares change w/ C?

– Evaluate sign of $\frac{\partial s_i}{\partial C}$ for $i \in \{a, m, s\}$

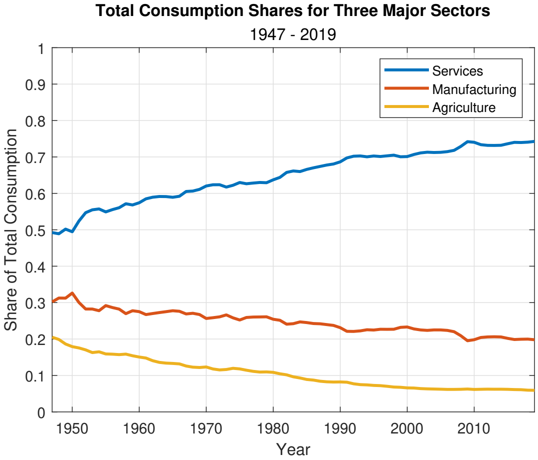
$$\frac{\partial\left(\frac{p_a c_a}{C}\right)}{\partial C} = \frac{(\omega_m + \omega_s)p_a\bar{c}_a - \omega_a p_s \bar{c}_s}{(\omega_a + \omega_m + \omega_s)C^2} < 0$$

$$\frac{\partial\left(\frac{p_s c_s}{C}\right)}{\partial C} = \frac{(\omega_a + \omega_m)p_s\bar{c}_s - \omega_s p_a \bar{c}_a}{(\omega_a + \omega_m + \omega_s)C^2} > 0$$

$$\frac{\partial\left(\frac{p_m c_m}{C}\right)}{\partial C} = \left(\frac{-\omega_m}{\omega_a + \omega_m + \omega_s}\right)\left(\frac{p_a\bar{c}_a + p_s\bar{c}_s}{C^2}\right) \lesseqgtr 0$$

Problem 2 (cont.)

(c) Plot expenditure shares using BEA data.



Problem 2 (cont.)

(d) Calibration

- Assume $\omega_a + \omega_m + \omega_s = 1 \qquad p_a = p_m = p_s = 1 \qquad \bar{c}_m = 0$
- Use 2019 shares from (c) to define ω_i for $i \in \{a, m, s\}$

$$\omega_a = 0.0592 \qquad \omega_m = 0.1981 \qquad \omega_s = 0.7426$$

- Find $\bar{c}_a$, $\bar{c}_s$ s.t. model matches 1947 shares (tolerance ± 0.5 percent)

$$\bar{c}_a \simeq -23 \qquad \bar{c}_s \simeq 121$$

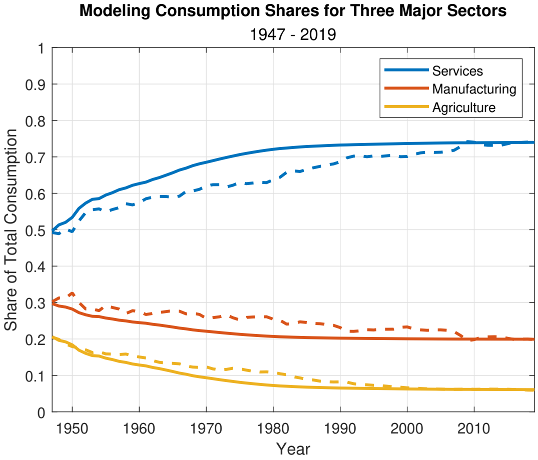
Problem 2 (cont.)

(e) Interpret values of $\bar{c}_a$, $\bar{c}_s$

- Shift marginal utility of consumption
- $\bar{c}_a < 0 \implies$ ag. goods are **necessities**
- $\bar{c}_s > 0 \implies$ services are **luxuries**
- The income elasticity of agricultural goods is relatively inelastic compared to the income elasticity of services

Problem 2 (cont.)

(f) Simulate expenditures path in the model.



Problem 3

- Use Newton's method to solve two systems of equations

(a) One equation, one unknown w/ clear analytical solution ($x = 3$)

(b) No clear analytical solution for second system

- Multiple ways to obtain Jacobian in Matlab (Symbolic Math toolbox)
- I found it easiest to "hard-code" Jacobian

$$\mathcal{J} = \begin{bmatrix} 1 - x_2 & 1 - x_1 \\ \exp(-x_2) & -x_1 \exp(-x_2) \end{bmatrix}$$

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J = [1-x2, 1-x1; exp(-x2), -x1*exp(-x2)]
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Problem 3 (cont.)

- Advantages of Newton's Method:
  - Fast
  - Can work for any system of nonlinear equations
- Disadvantages:
  - Requires derivatives
  - Jacobian must be invertible
  - Usually requires a good initial guess

General Overview

- **Six problems** related to Solow Growth Model
- Refer to lecture slides
- Problem 2 asks for several proofs
  - Write down what you're allowed to assume about $f(k)$
  - It wouldn't be assumed if you didn't need it to write the proof
  - Not all assumed properties of $f(k)$ are needed for every proof
- Problem 3 requires Penn World Tables 10.01
  - Okay to use Excel, or CSV format in program of your choice

Problem Set 2 Hints

1. Basic Solow model
   a) Use provided aggregate equations and write law of motion of capital in per-capita terms
   b) Impose steady state condition on equation from (a)
   c) Make plot (by hand or computer) with t on x-axis and Y_t on y-axis
   d) Compare steady state capitals of two economies

Problem Set 2 Hints (cont.)

2. Write proofs using assumed properties of $f(k)$ on p. 4 of lecture slides. You may also assume properties from other parts of problem are true.
   a) Evaluate sign of $h'(k)$
   b) Write $h'(k)$ again. What happens as $k \to 0$?
   c) Write $h'(k)$ again. What happens as $k \to \infty$?
   d) Evaluate sign of $h''(k)$
   e) Steady state capital k^* satisfies $h(k^*) = k^*$. **Why is it unique?**
   f) Notice that k^* is one such $\bar{k}$
   g) There are three cases to show
      - Part (e): $k_0 = k^*$
      - Part (f): $k_0 > k^*$
      - Also show $k_0 < k^*$

Problem Set 2 Hints (cont.)

3. Decompose cross-country differences in income between differences in savings rate and differences in productivity
   - Download Penn World Tables 10.01
4. Analyze model-implied differences in savings rates that would explain income differences between U.S. and poorer countries
   a) Find an equation for $y(s)/y(s = 0.2)$
   b) Set $y(s)/y(s = 0.2) = 1/30$ and solve s in terms of α
   c) Plug $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ into $s(\alpha)$

Problem Set 2 Hints (cont.)

5. Analyze how TFP and α affect transition paths of the economy
   a) Produce table of # years for economy to accumulate capital

	α				
Transition	0.1	0.3	0.5	0.7	0.9
$0.1k^*-0.6k^*$					
$0.6k^*-0.8k^*$					
$0.8k^*-0.9k^*$					
$0.9k^*-0.95k^*$					
$0.95k^*-0.975k^*$					

Problem Set 2 Hints (cont.)

b) Fix $\alpha = 0.5$. Allow TFP to vary and produce another table

	A		
Transition	1	2	4
$0.1k^*$–$0.6k^*$			
$0.6k^*$–$0.8k^*$			
$0.8k^*$–$0.9k^*$			
$0.9k^*$–$0.95k^*$			
$0.95k^*$–$0.975k^*$			

- Note: Copy values for column $A = 1$ from part (a) results

c) Discuss results.

6. Consider affine (i.e., not concave) production function

Problem Set 3

- Due **Friday, Sept. 26 at 11:59 p.m.**
- Two **dynamic optimization** problems
  1. Finite-horizon planner problem
  2. Finite-horizon consumption-saving problem
- Both “dynamic” due to **capital/asset accumulation**
  - Today’s savings determine resources for future consumption
  - **Euler equation** characterizes optimal intertemporal balance

Problem 1: Finite-horizon planning problem

(a) Solve the optimal savings path

- Write FOC wrt planner's choice k_{t+1}
- Write equation w/ $z_{t+1} \equiv \frac{k_{t+2}}{Ak_{t+1}^{\alpha}}$ on left side and z_t on right side
- Write z_t in terms of α, β, and z_{t+1}
- Solve z_t in terms of α, β, t, and T by **backwards induction**
  - Using $z_T = 0$ (**why is this true?**), solve z_{T-1}
  - Use z_{T-1} to find z_{T-2}, and so on
  - Recognize pattern to write z_t
- Write $k_{t+1} = z_t \times Ak_t^{\alpha}$

(b) Evaluate and discuss $\lim_{T\to\infty} z_0$, $\lim_{T\to\infty} z_1$

- What is happening to share of output that is saved in the long-run?

Problem 2: Consumption-savings problem

(a) Derive the Euler equation
   - Combine FOCs wrt c_t, c_{t+1}, and a_{t+1}

(b) Show **intertemporal elasticity of substitution** (IES) equals $1/\sigma$
   - Use Euler equation to write $\log(c_{t+1}/c_t)$
   - Evaluate $\partial \log\left(c_{t+1}/c_t\right)/\partial \log(1+r)$
   - What does the IES represent?

Problem 2: Consumption-savings problem (cont.)

(c) Find optimal consumption in terms of intial assets a_0 and endowments e_t

- Derive **lifetime budget constraint**
  - Combine period budget constraints through savings decisions
  - Find equation relating P.V. consumption, a_{T+1} to P.V. of endowments, a_0
  - Note that $a_{T+1} = 0$ (for the same reason $z_T = 0$ in #1)
- Find c_0
  - Redefine consumer problem as preference max. s.t. lifetime b.c.
  - Find Euler using FOCs wrt c_t, c_{t+1}
  - Use Euler to write c_t in terms of c_0, β, r, and σ
  - Replace c_t in lifetime budget constraint to solve c_0
    (You need to evaluate finite geometric series)
- You already wrote c_t in terms of c_0, so you're finished!