TA Session 5

Michael Kotrous
John Munro Godfrey, Sr. Department of Economics
University of Georgia

ECON 8040: Macroeconomics I
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Today's Session

- Review Problem Set 0 solutions
- **Problem Set 1** grades on eLC
- **Computation Exercise 1** to be graded soon
- **Problem Set 2** due Friday, Sept. 19 at 11:59 p.m.

Grading Comments

- General rules for good data visualization
  - **Time series** displayed as *lines*
  - **Cross-section** displayed as *scatter plot*
  - Titles, labels should be **human-readable words**, not machine-readable variables
  - Try your best to align plots and text near one another in LaTeX

Remarks on #9

- Keep CE definition separate from work of solving it
  - Don't include extraneous variables (e.g., no capital in this problem)
  - Don't write FOCs in CE definition in part (a)
  - Don't write answers in part (b) w/out showing some work
- Broad structure of CE definition
  - "CE is prices, policy, and allocations such that ..."
  - Given prices and policy, HH allocation maximizes preferences
  - Given prices and policy, firm maximizes profit
  - Government (if applicable)
  - Markets clear

Kaldor Facts

Set of **stylized facts** that motivated neoclassical macroeconomists

1. Output per capita has grown at ≈ constant rate (≈ 2%) ▸ Plot
2. Capital-output ratio ≈ constant (and ≈ 3) ▸ Plot
3. Consumption-output ratio ≈ constant ▸ Plot
4. Wage growth ≈ per capita output growth ▸ Plot
5. Real interest rate ≈ constant (in long-run) ▸ Plot
6. Labor share of income ≈ constant ▸ Plot
7. Hours worked per capita ≈ constant past 50 years ▸ Plot

Problem 1

National Income Accounting Definitions

- **National Income:** Total income of all Americans
  - Wages and salaries paid to workers
  - Proprietor's income
  - Corporate profits
- **Gross National Product:** Total income earned by nation's factors of production, *regardless of location*

$$\text{GNP} = \text{National Income} + \text{Depreciation} + \text{Sales Tax}$$

- **Gross Domestic Product:** Total income earned by factors of production *within nation's borders, regardless of nationality*

$$\text{GDP} = \text{GNP} - \text{factor payments } \textit{from} \text{ abroad} + \text{factor payments } \textit{to} \text{ abroad}$$

Problem 1

a) Expenditures Approach

$$\text{GDP} = C + I + G + NX = 24.38 \text{ trillion}$$

c) Income Approach

$$\begin{aligned}\text{GDP} =& \underbrace{\text{GNI} + \text{Sales Tax} + \text{Depreciation}}_{\text{GNP}} \\ &+ \text{Payments } \textit{to} \text{ abroad} - \text{Payments } \textit{from} \text{ abroad} \\ =& 25.23 \text{ trillion}\end{aligned}$$

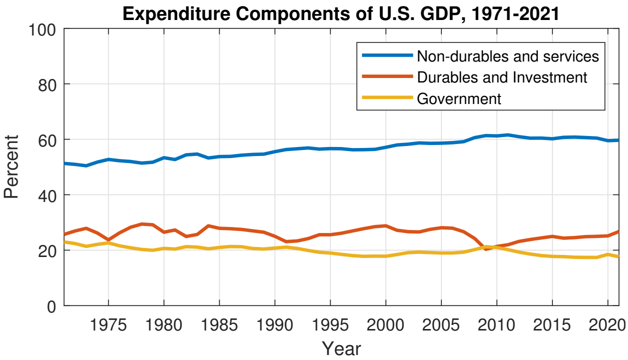
- Gap is due to discrepancies in measurement

Problem 1

b) National income

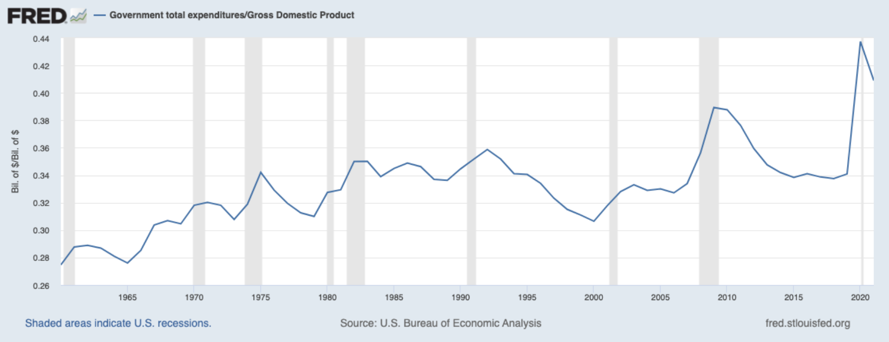
$$
\begin{aligned}
\text{GNI} =& \text{Compensation of Employees} + \text{Net Interest} \\
& + \text{Net Business Transfers} + \text{Rental Income} + \\
& + \text{Proprietor's Income} + \text{Corp. Profits} \\
& + \text{Current Surplus of Gov. Ent.} \\
=& 19.76 \text{ trillion}
\end{aligned}
$$

Problem 2, parts a,b,c



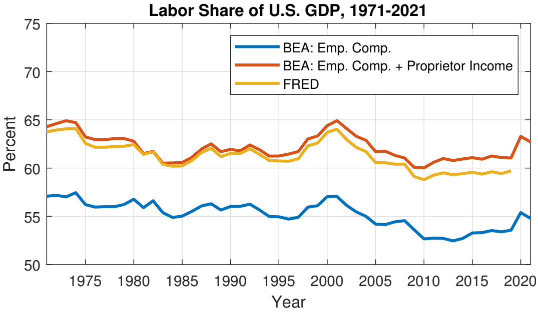
- Should sum close to 1 (GDP $\approx C + I + G$ when NX small)

Problem 2, part c



- Alternative calculation: Gov't Share $\equiv \frac{\text{Gov't Total Exp.}}{\text{GDP}}$
- Gov't share is **counter-cyclical**

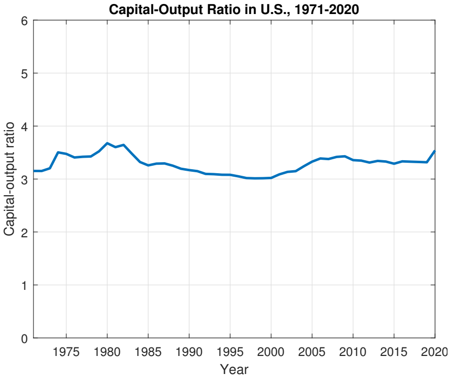
Problem 2, part d



- **KF # 6:** Labor share of income ≈ constant
- Recent decline spurred research:
  - "Superstar Firms" (Autor et. al, 2020)
  - "Rise of Market Power" (De Loecker, Eeckhout, and Unger, 2020)

Kaldor Facts

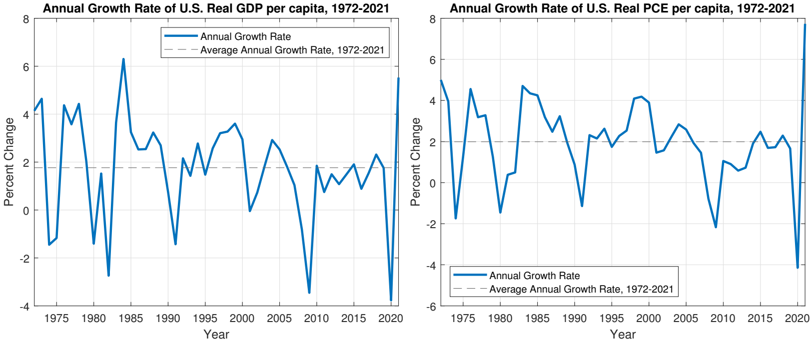
Problem 2, part e



- **KF #2:** Capital-output ratio $\approx$ constant (and $\approx$ 3)

◂ Kaldor Facts

Problem 2, parts f,g

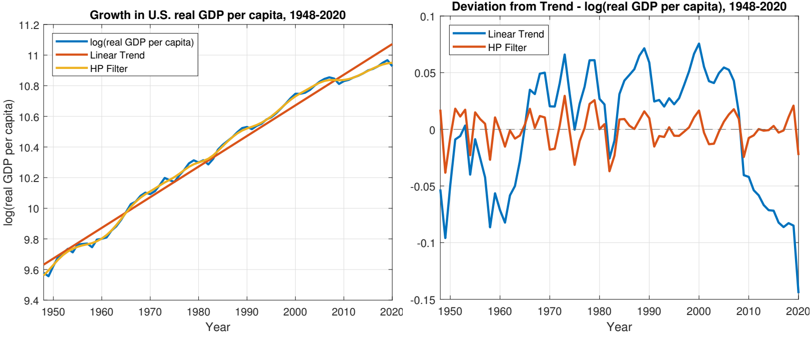


Problem 3

Report trends and residuals for data series:

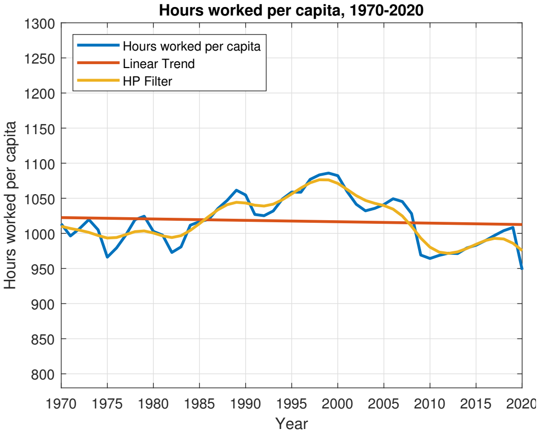
1. Linear trend
   a) Compute $\log(x)$
   b) Matlab's `detrend` function fits line and returns residuals $\hat{e}$
   c) Trend $\widehat{\log(x)} = \log(x) - \hat{e}$
2. HP Filter
   - Compute $\log(x)$
   - Matlab's `hpfilter` function returns trend and residuals

Problem 3, part a



- Hodrick-Prescott Filter does better job capturing business cycle (NBER business cycle dates)

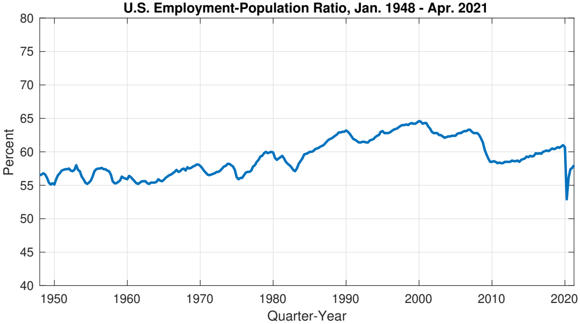
Problem 3, part d



- **KF #7:** Hours worked per capita ≈ constant past 50 years

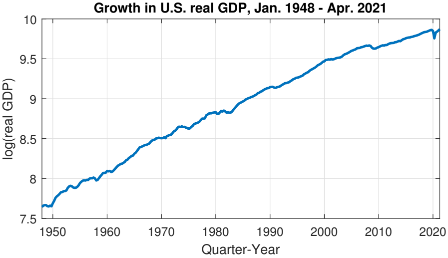
◂ Kaldor Facts

Problem 4, part a



- What do you observe from 1960–2000? Why? For research on post-2000 decline, see Abraham & Kearney (2020)
- In employment statistics, “population” is short for **adult, civilian, non-institutional population** (*not* “total population”)

Problem 4, part b



- **KF #1:** Output per capita has grown at ≈ constant rate (≈ 2%)
- Why we interpret trend in natural log as "growth rate":

$$\frac{d}{dx}\log(x) = \frac{1}{x} \Leftrightarrow d\log(x) = \frac{dx}{x} \equiv \%\Delta x$$

Kaldor Facts

Problem 5

a) From firm profit maximization,

$$r_t = \frac{\partial Y_t}{\partial K_t} = \alpha A_t \left(\frac{K_t}{H_t}\right)^{\alpha-1} = \alpha \frac{Y_t}{K_t}$$

$$w_t = \frac{\partial Y_t}{\partial H_t} = (1-\alpha) A_t \left(\frac{K_t}{H_t}\right)^{\alpha} = (1-\alpha)\frac{Y_t}{H_t}$$

b)

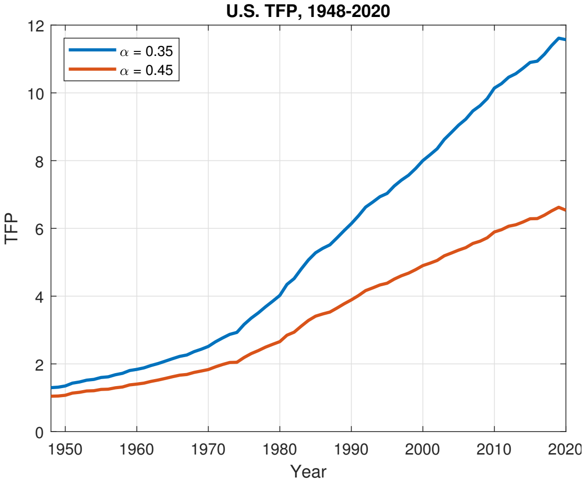
$$\frac{w_t H_t}{Y_t} = \frac{(1-\alpha)A_t K_t^{\alpha} H_t^{1-\alpha}}{A_t K_t^{\alpha} H_t^{1-\alpha}} = 1-\alpha$$

c) Labor share $= 1-\alpha \Leftrightarrow \alpha = 1 -$ Labor share ≈ 0.38

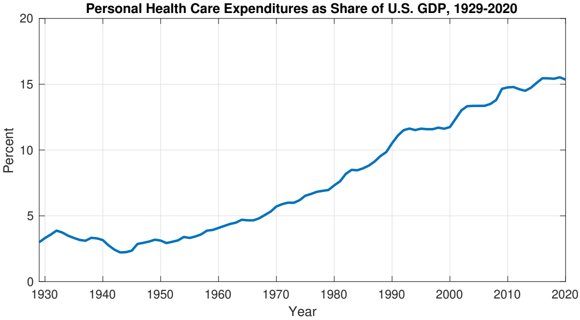
- $\frac{\text{Compensation of Employees + Proprietor's Income}}{\text{U.S. GDP}}$ (BEA)
- Labor Share of GDP (FRED)

Problem 5

d) $Y_t = A_t K_t^{\alpha} H_t^{1-\alpha} \Leftrightarrow A_t = \frac{Y_t}{K_t^{\alpha} H_t^{1-\alpha}}$

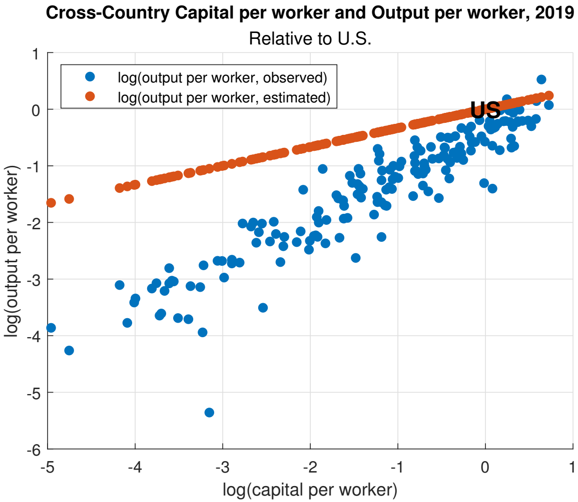


Problem 6



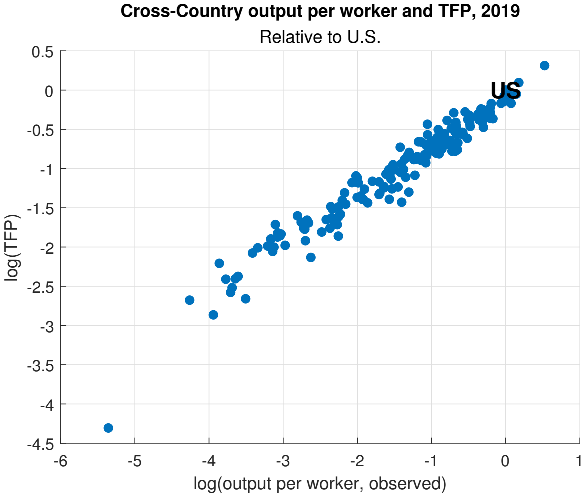
- Claim: "Health expenditures share has increased because the U.S. healthcare system is broken. Policy should limit spending growth."
- Response: "Spending a larger share of income on health as income grows is perfectly rational!" (Hall and Jones, 2007)

Problem 7, part a



Problem 7, part b

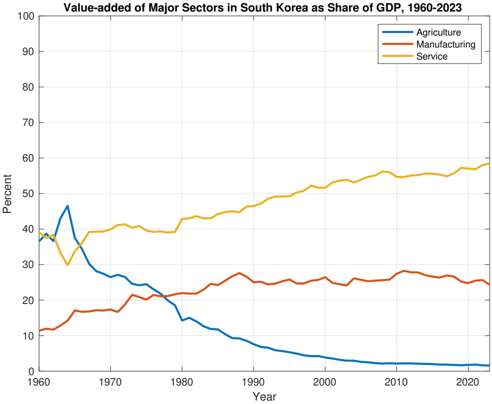
- If model were perfect, $y_i = k_i^\alpha \Rightarrow A_i = 1 \Rightarrow \log(A_i) = 0$



Problem 7, part c

- $\text{success}_1 \approx 0.16$; $\text{success}_2 \approx 0.19$
- $\Rightarrow$ Differences in capital per worker are insufficient to explain cross-country differences in output per worker
- $\Rightarrow$ “The secret sauce” TFP matters a lot

Problem 8



- South Korea is considered **growth miracle**
- As economies develop, $\downarrow$ ag share + $\uparrow$ services

Problem 9(a)

Competitive equilibrium is wage w^*, profit π^*, household allocation $\{c^{1*}, h^{1*}, c^{2*}\}$, and firm allocation $\{y^*, n^*\}$ such that:

1. Given wage w^*, household of type 1 makes allocation $\{c^{1*}, h^{1*}\}$ that solves utility maximization:

$$\max_{c^1, h^1} (1-\phi)\log(c^1) + \phi \log(1-h^1)$$

subject to $c^1 = wh^1, c^1 \geq 0, 0 \leq h^1 \leq 1$

Problem 9(a)

2. Given firm profit π^*, household of type 2 makes allocation c^{2*} that solves utility maximization:

$$\max_{c^2} (1-\phi)\log(c^2)$$

subject to $c^2 = \pi, c^2 \geq 0$

Problem 9(a)

3. Given wage w^*, firm allocation $\{y^*, n^*\}$ maximizes profit:

$$\max_{y,n} y - wn$$

subject to $y = \frac{n^\gamma}{\gamma}$

4. Markets clear
   - Consumption good: $c^{1*} + c^{2*} = y^* \leftarrow$ GDP
   - Labor market: $h^{1*} = n^*$

Problem 9(b)

1. By worker's ($i = 1$) first-order conditions, marginal rate of transformation (MRT) equals marginal rate of substitution (MRS):

$$\underbrace{w^*}_{\text{MRT}} = \underbrace{\frac{\phi}{1-\phi} \cdot \frac{c^{1*}}{1-h^{1*}}}_{\text{MRS}} \tag{1}$$

2. Firm's first-order condition (FOC): $w^* = (n^*)^{\gamma-1}$
   - For optimal labor demand, **marginal product of labor** equals wage
3. Replace firm FOC, labor market clearing condition, and worker's budget constraint into (1) to solve labor supply

$$h^{1*} = 1 - \phi \qquad \leftarrow \text{Intuition?}$$

Problem 9(b)

4. Replace h^{1*} in equations for labor demand, wage, production function, worker consumption, and profit to finish:

$$n^* = 1 - \phi$$

$$w^* = (1 - \phi)^{\gamma - 1} \qquad c^{1*} = w^* h^{1*} = (1 - \phi)^{\gamma}$$

$$y^* = \frac{(1 - \phi)^{\gamma}}{\gamma} \qquad c^{2*} = \pi^* = y^* - w^* n^* = \frac{(1 - \gamma)(1 - \phi)^{\gamma}}{\gamma}$$

General Remarks

- When defining CE, state what the household, firm takes **as given**
  - Determines what agents treat as *constant* when optimizing

Example: PS1 Problem 3

- Firm FOC: $w = A_m$ Gov't budget: $T = \tau w H_m$
- HH problem:

$$\max_{C_m, C_h, H_m, H_h, L} \left[\alpha C_m^{\frac{\sigma-1}{\sigma}} + (1-\alpha) C_h^{\frac{\sigma-1}{\sigma}} \right] + \log L$$

st $C_m = (1-\tau) A_m H_m + \tau A_m H_m$ $C_h = A_h H_h$ $H_m + H_h + L = 1$

- What is wrong with this setup?

General Remarks (cont.)

- When defining CE, state what the household, firm takes **as given**
  - Determines what agents treat as *constant* when optimizing

Example: PS1 Problem 3

- Firm FOC: $w = A_m$ Gov't budget: $T = \tau w H_m$
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st $C_m = (1-\tau)A_m H_m + \tau A_m H_m$ $C_h = A_h H_h$ $H_m + H_h + L = 1$

- When household is *price-taker*, they do not internalize effect of their choices on equilibrium wage! They also take policy τ, T as given!

General Remarks (cont.)

- When we assume price-taking / policy-taking behavior, we
  1. Write HH optimality conditions w/ prices, policies treated as **scalars**
  2. Take FOCs, *then* replace prices with firm optimality conditions, which relate prices to household allocations through market-clearing
- Does household make different choices if we replace factor price equations first?
  - Yes! Illustrative example is model from **PS0 Problem 9**

Price-Taking Assumption: PS0 Problem 9

- When household takes wage *as given*, we showed

$$h^{1*} = 1 - \phi \qquad c^{1*} = (1 - \phi)^{\gamma}$$

- Does household change labor supply if it knows its effect on wage?
  - Solve equilibrium when household knows **inverse labor demand**
  - By firm FOC

$$w(h_m^1) = (h_m^1)^{\gamma - 1}$$

  - Household is "monopoly" seller of labor, so denote w/ m subscript

Price-Taking Assumption: PS0 Problem 9 (cont.)

- Taking $w(h_m^1)$ as given, type 1 preference maximization problem is:

$$\max_{c_m^1, h_m^1} (1-\phi)\log(c_m^1) + \phi \log(1-h_m^1)$$

s.t. $c_m^1 = w(h_m^1)h_m^1 \qquad c_m^1 \geq 0 \qquad 0 \leq h_m^1 \leq 1$

- Household's optimality condition is

$$\underbrace{\gamma(h_m^1)^{\gamma-1}}_{MRT} = \underbrace{\frac{\phi}{1-\phi} \times \frac{c_m^1}{1-h_m^1}}_{MRS}$$

  - Notice MRT depends on h_m^1 (was scalar w under price-taking)

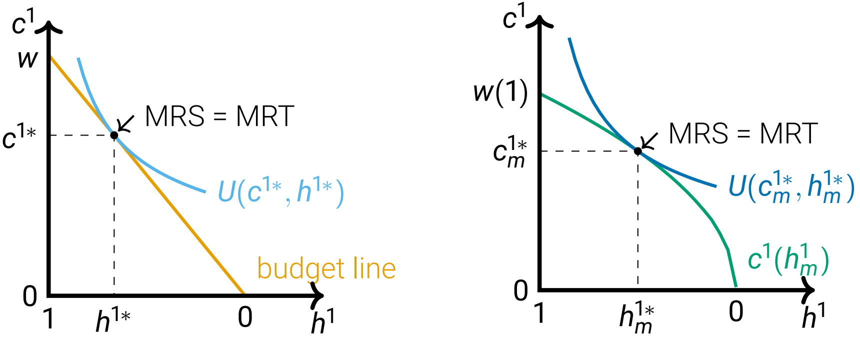
Price-Taking Assumption: PS0 Problem 9 (cont.)

- Replacing $c_m^1 = w(h_m^1)h_m^1$ in optimality condition, you can show

$$h_m^{1*} = \underbrace{\left(\frac{\gamma}{\phi + \gamma(1-\phi)}\right)}_{\mu \in (0,1)}(1-\phi)$$

- Thus, $h_m^{1*} < h^{1*}$. Why?
  - Wage is *decreasing* in labor supply
  - If worker decreases time spent working, $\uparrow$ wage
  - Household can gain lot of leisure and give up only small amount of consumption, increasing utility

Price-Taking Assumption: PS0 Problem 9 (cont.)



(a) Price-Taking **(b)** Pricing Power

Figure: Optimal Choice of Consumption and Labor Supply – Type 1 HH

Price-Taking Assumption: PS0 Problem 9 (cont.)

- Suppose $\phi = 1/3$ and $\gamma = 1/2$

Variable	**Price-Taking**	**Pricing Power**	**Change**
Wage	1.2247	1.4142	$\uparrow$
Leisure	0.3333	0.5000	$\uparrow$
Consumption	0.8165	0.7071	$\downarrow$
Utility	-0.5014	-0.4621	$\uparrow$

Price-Taking Assumption: PS0 Problem 9 (cont.)

- It matters how we write price equations in household problem
  - Write price as scalar $\Longrightarrow$ **price-taking**
  - Write price as function of allocations $\Longrightarrow$ **pricing power**
- As in principles, pricing power vs. price-taking affects optimal choices by households and firms

Problem 1

(a) CE is household allocation $\{c_i, h_i, k_i\}_{i\in[0,1]}$, firm allocation (y, n, k), and prices (w, r) such that

1) Given prices and endowment $\bar{k}_i = 1$, household i's allocation solves utility maximization:

$$\max_{c_i, h_i} u(c_i) - v(h_i)$$

subject to $c_i \leq wh_i + rk_i$, where $h_i \in \{0, 1\}$ and $k_i = \bar{k}_i$

2) Given prices, firm allocation solves profit maximization:

$$\max_{y,n,k} y - rk - wn$$

subject to $y = k^\alpha n^{1-\alpha}$

Problem 1 (cont.)

3) The markets are clearing
   - Consumption good: $y = \int_0^1 c_i \, di$ $\leftarrow$ **GDP**
   - Labor: $n = \int_0^1 h_i \, di$
   - Capital: $k = \int_0^1 \bar{k}_i \, di = 1$

Problem 1 (cont.)

(b) Solve aggregate equilibrium allocation, wage and rental rate

- Indifference between working and not working in equilibrium

$$\underbrace{\log(w+r)-\log(7)}_{\text{working}} = \underbrace{\log(r)}_{\text{not working}}$$
$$\Rightarrow \frac{w}{r} = 6$$

- Firm's first-order conditions give w, r in terms of n
- Replace for w, r in ratio above, solve n, then r, w

$$n = \frac{1}{3} \qquad r = \left(\frac{1}{3}\right)^{5/3} \qquad w = 2\left(\frac{1}{3}\right)^{2/3}$$

Problem 2

(a) CE is allocation by type 1 household (c_1^1, c_2^1), type 2 household (c_1^2, c_2^2) and price q such that

1) Given q, consumer 1's allocation solves utility maximization:

$$\max_{c_1^1, c_2^1} (1-\beta)u(c_1^1) + \beta u(c_2^1)$$

subject to $c_1^1 + qc_2^1 = 1$

2) Given q, consumer 2's allocation solves utility maximization:

$$\max_{c_1^2, c_2^2} (1-\beta)u(c_1^2) + \beta u(c_2^2)$$

subject to $c_1^2 + qc_2^2 = q$

3) Markets clear: $c_1^1 + c_1^2 = 1 \qquad c_2^1 + c_2^2 = 1$

Problem 2 (cont.)

(b) Suppose $\beta = 0.5$. What is equilibrium price q?

- First-order condition for type i

$$c_1^i = q^{1/\sigma} c_2^i \tag{2}$$

- Use MCCs and (2) to find q

$$c_1^1 + c_1^2 = q^{1/\sigma}(c_2^1 + c_2^2) \Leftrightarrow q = 1$$

Problem 2 (cont.)

(c) Assume $\sigma = 1$ and $\beta \in (0, 1)$. Solve allocations and price q

1) First order condition for type i
2) Use FOC and MCCs to find q

$$q = \frac{\beta}{1 - \beta}$$

3) Use FOC and budget constraints to find allocations

$$c_1^1 = c_2^1 = 1 - \beta$$
$$c_1^2 = c_2^2 = \beta$$

- How do price and allocation vary with β?

Problem 3

(a) TDCE is HH allocation $(C_m, C_h, L, H_m^s, H_h, Y_h)$, firm allocation (Y_m, H_m^d), policy (τ, T) and wage w such that:

1) Given wage w and policy (τ, T), HH allocation solves UMP:

$$\max_{C_m, C_h, L, H_m^s, H_h, Y_h} \alpha C_m^{\frac{\sigma-1}{\sigma}} + (1-\alpha) C_h^{\frac{\sigma-1}{\sigma}} + \log L$$

subject to

$$C_m = H_m^s w(1-\tau) + T$$
$$C_h = Y_h = A_h H_h$$
$$H_m + H_h + L = 1$$

Problem 3 (cont.)

2) Given wage and policy, firm maximizes profit s.t. production function
3) Government budget balances: $T = \tau w H_m^s$
4) Markets clear
   - Market Goods: $Y_m = C_m$
   - Labor: $H_m^s = H_m^d$
   - No market for home-produced good!

Problem 3 (cont.)

(b) Find the ratio of H_m and H_h in terms of parameters and taxes rates

$$\frac{H_m}{H_h} = \left(\frac{A_h}{A_m}\right)^{1-\sigma} \times \left(\frac{\alpha}{1-\alpha} \times (1-\tau)\right)^{\sigma}$$

- What happens to H_h as A_h increases (relative to A_m)?

$$\frac{\partial\left(\frac{H_m}{H_h}\right)}{\partial\left(\frac{A_h}{A_m}\right)} = (1-\sigma) \times \left(\frac{A_h}{A_m}\right)^{-\sigma} \times \left(\frac{\alpha}{1-\alpha} \times (1-\tau)\right)^{\sigma}$$

  - H_h increases if $\sigma > 1$ (i.e., home, market good are substitutes)
  - H_h decreases if $\sigma < 1$ (i.e., home, market good are complements)

Problem 3 (cont.)

(c) Solve leisure L in terms of H_h, parameters and tax rate. What happens to leisure as H_h falls?

$$L = 1 - H_h - H_h \times \left(\frac{A_h}{A_m}\right)^{1-\sigma} \times \left(\frac{\alpha}{1-\alpha} \times (1-\tau)\right)^{\sigma}$$

(d) Suppose you want to comment on the effect on improvement in home appliances on labor supply and leisure. How would you use this model to make that comment?

- Effect on labor supply depends on whether home, market good being substitutes or complements
- Leisure increases

Problem 3 (cont.)

- Over the past century,
  - Women's labor supply increased but was offset by declines in men's labor supply
  - Time in home production decreased for women but was offset by increases in men's time in home production
  - Per capita leisure increased four hours per week
- Source: Ramey and Francis (2009), "A Century of Work and Leisure," *AEJ: Macro*

Problem 4

(a) TDCE is household allocation (c, h, k^s), firm allocation (y, n, k^d), prices (w, r), and policy (τ, g, B) such that

1) Given prices and policy, consumer allocation solves UMP
2) Given prices and policy, firm allocation maximizes profit
3) Government budget balance: $\tau wh = g + B$
4) Markets clear
   - Consumption good: $y = c + g$
   - Labor: $h = n$
   - Capital: $k^s = k^d$

Problem 4 (cont.)

(b) Find ϕ such that model matches U.S. labor supply at its tax rate

- Use HH FOCs, firm's FOC wrt n, and MCCs to get ϕ in terms of data, α
- Hint #1: You can write wage in terms of y, n
- Hint #2: c/y is a number

$$\frac{\phi}{1-\phi} = \frac{0.8h}{(100-h)(1-\tau)(1-\alpha)}$$

$$\phi \simeq 0.4114$$

Problem 4 (cont.)

(c) Now suppose the US adopts the tax rate in each country in the table. Find the predicted labor supply $\widehat{h}$.

- Using household FOCs, write h in terms of ϕ, τ, and α
- Use $\alpha = 1/3$, $\phi \simeq 0.4114$, and τ in table to find predicted labor supply $\widehat{h}$ for each country

Problem 4 (cont.)

Table: Labor Supply

		Labor Supply	
Country	Tax Rate	Actual	Predicted
Germany	0.59	19.3	19.27
France	0.59	17.5	19.27
Italy	0.64	16.5	17.33
Canada	0.52	22.9	21.84
U.K	0.44	22.8	24.59
Japan	0.37	27.0	26.84
U.S.	0.40	25.9	25.90

Problem 5

(a) Assuming Cobb-Douglas technology, show wage, rental rate depend only on capital per worker.

- Define profit maximization problem: Given w_t, r_t, firm allocation k_t, n_t solves

$$\max_{k_t, n_t} A k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - r_t k_t$$

- By FOCs,

$$r_t = \alpha A \left(\frac{k_t}{n_t} \right)^{\alpha - 1}$$
$$w_t = (1 - \alpha) A \left(\frac{k_t}{n_t} \right)^{\alpha}$$

Problem 5 (cont.)

(b) Write w_t, r_t in terms of output per unit of labor / capital, respectively

- Using result from (a),

$$r_t = \alpha A \left(\frac{k_t}{n_t}\right)^{\alpha-1} \times \frac{k_t}{k_t} = \frac{\alpha y_t}{k_t}$$

$$w_t = (1-\alpha) A \left(\frac{k_t}{n_t}\right)^{\alpha} \times \frac{n_t}{n_t} = \frac{(1-\alpha) y_t}{n_t}$$

Problem 5 (cont.)

(c) Repeat (a) and (b) for CES technology

- Show (b) directly from FOCs of profit-maximization

$$r_t = \left(\frac{\alpha y_t}{k_t}\right)^{1/\sigma}$$
$$w_t = \left(\frac{(1-\alpha) y_t}{n_t}\right)^{1/\sigma}$$

- Prove $y_t = f(k_t, n_t) = n_t \times f(k_t/n_t, 1)$. Then show (a)

$$r_t = \left(\frac{\alpha f(k_t/n_t, 1)}{k_t/n_t}\right)^{1/\sigma}$$
$$w_t = [(1-\alpha) f(k_t/n_t, 1)]^{1/\sigma}$$

Problem 5 (cont.)

(d) What happens to capital, labor demand as σ changes?

- $\sigma \to 0 \implies$ **perfect complements**
  - Firm demands capital, labor in fixed proportion that depends on α
- $\sigma \to \infty \implies$ **perfect substitutes**
  - Firm demands *only* capital or *only* labor (depending on relative price)
  - If relative prices are equal, firm demands any combo of capital, labor

General Overview

- **Six problems** related to Solow Growth Model
- Refer to lecture slides
- Problem 2 asks for several proofs
  - Write down what you're allowed to assume about $f(k)$
  - It wouldn't be assumed if you didn't need it to write the proof
  - Not all assumed properties of $f(k)$ are needed for every proof
- Problem 3 requires Penn World Tables 10.01
  - Okay to use Excel, or CSV format in program of your choice

Problem Set 2 Hints

1. Basic Solow model
    a) Use provided aggregate equations and write law of motion of capital in per-capita terms
    b) Impose steady state condition on equation from (a)
    c) Make plot (by hand or computer) with t on x-axis and Y_t on y-axis
    d) Compare steady state capitals of two economies

Problem Set 2 Hints (cont.)

2. Write proofs using assumed properties of $f(k)$ on p. 4 of lecture slides. You may also assume properties from other parts of problem are true.
   a) Evaluate sign of $h'(k)$
   b) Write $h'(k)$ again. What happens as $k \to 0$?
   c) Write $h'(k)$ again. What happens as $k \to \infty$?
   d) Evaluate sign of $h''(k)$
   e) Steady state capital k^* satisfies $h(k^*) = k^*$. **Why is it unique?**
   f) Notice that k^* is one such $\bar{k}$
   g) There are three cases to show
      - Part (e): $k_0 = k^*$
      - Part (f): $k_0 > k^*$
      - Also show $k_0 < k^*$

Problem Set 2 Hints (cont.)

3. Decompose cross-country differences in income between differences in savings rate and differences in productivity
   - Download Penn World Tables 10.01
4. Analyze model-implied differences in savings rates that would explain income differences between U.S. and poorer countries
   a) Find an equation for $y(s)/y(s=0.2)$
   b) Set $y(s)/y(s=0.2)=1/30$ and solve s in terms of α
   c) Plug $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ into $s(\alpha)$

Problem Set 2 Hints (cont.)

5. Analyze how TFP and α affect transition paths of the economy
   a) Produce table of # years for economy to accumulate capital

	α				
Transition	0.1	0.3	0.5	0.7	0.9
$0.1k^*-0.6k^*$					
$0.6k^*-0.8k^*$					
$0.8k^*-0.9k^*$					
$0.9k^*-0.95k^*$					
$0.95k^*-0.975k^*$					

Problem Set 2 Hints (cont.)

b) Fix $\alpha = \mathbf{0.5}$. Allow TFP to vary and produce another table

	A		
Transition	1	2	4
$0.1\boldsymbol{k}^* - 0.6\boldsymbol{k}^*$			
$0.6\boldsymbol{k}^* - 0.8\boldsymbol{k}^*$			
$0.8\boldsymbol{k}^* - 0.9\boldsymbol{k}^*$			
$0.9\boldsymbol{k}^* - 0.95\boldsymbol{k}^*$			
$0.95\boldsymbol{k}^* - 0.975\boldsymbol{k}^*$			

- Note: Copy values for column $A = \mathbf{1}$ from part (a) results

c) Discuss results.

6. Consider affine (i.e., not concave) production function

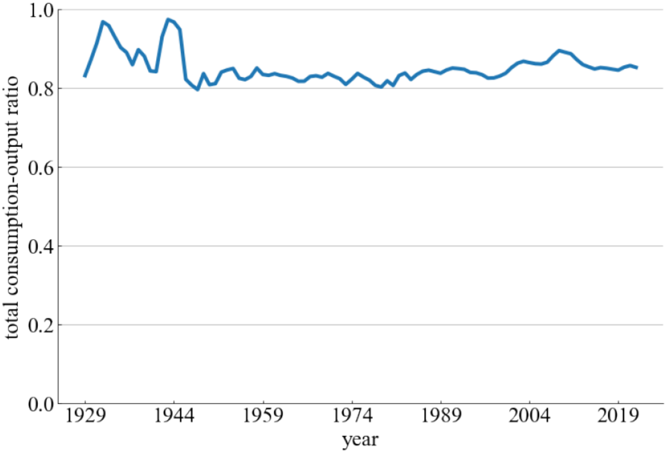


Figure: AKMM Figure 2.11: Ratio of total consumption to output

Kaldor Facts

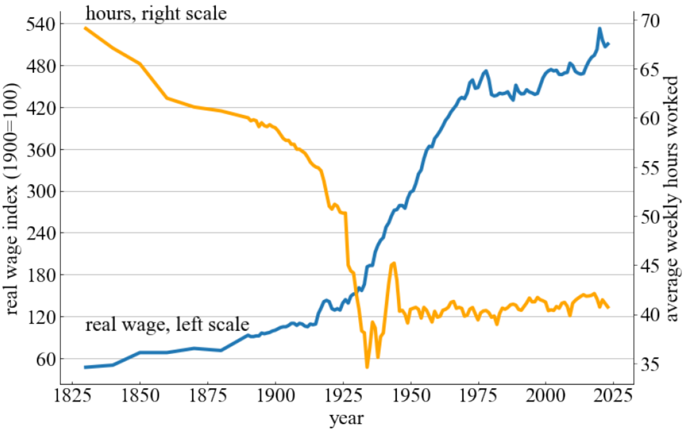


Figure: AKMM Figure 2.6: Real wages in U.S.

◂ Kaldor Facts

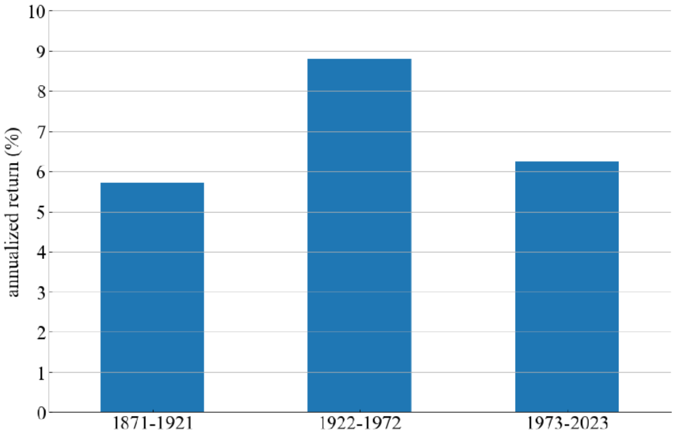


Figure: AKMM Figure 2.4: Return on capital

◂ Kaldor Facts