TA Session 4

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Today's Session

- Problem Set 1 due **Friday, Sept. 5 at 11:59 p.m.**
- Computation Exercise 1 due **Tuesday, Sept. 9 at 2:00 p.m.**

Grading Comments

- General rules for good data visualization
  - **Time series** displayed as *lines*
  - **Cross-section** displayed as *scatter plot*
  - Titles, labels should be **human-readable words**, not machine-readable variables
  - Try your best to align plots and text near one another in LaTeX

Remarks on #9

- Keep CE definition separate from work of solving it
  - Don't include extraneous variables (e.g., no capital in this problem)
  - Don't write FOCs in CE definition in part (a)
  - Don't write answers in part (b) w/out showing some work
- Broad structure of CE definition
  - "CE is prices, policy, and allocations such that . . . "
  - Given prices and policy, HH allocation maximizes preferences
  - Given prices and policy, firm maximizes profit
  - Government (if applicable)
  - Markets clear

Problem Set 1

Due Friday, Sept. 5 at 11:59 p.m.

- Five static models that you will solve analytically
- Write verbose competitive equilibrium definitions
  - State all equilibrium prices and allocations *first*
  - State clearly which equilbrium objects households / firms *take as given* and what they *choose*
  - Refer to TA2 slides on PS0, #9 for an example
- Generally ordered easiest to hardest
- If stuck, move on to the next problem
- If provided hints don't help, email me for additional hints

Problem 1

Economy w/ Indivisible Labor Supply

(a) Define competitive equilibrium

(b) In equilibrium, households are *indifferent* between working full-time and not working at all

1. Use this condition to write down an equation
   - Think carefully about how much workers/non-workers consume
2. Solve for $\frac{r^*}{w^*}$ (it equals a constant)
3. Write down firm's FOCs
4. Combine expression from steps 2 and 3 to write an equation that has equilibrium labor supply n^* as its only variable and solve

Problem 2

Two-Good Economy

(a) Define competitive equilibrium.

1) Consumer 1's allocation solves their optimization problem
2) Consumer 2's allocation solves their optimization problem
3) Markets clear

(b) Suppose $\beta = 0.5$. What is equilibrium price q?

1) Write Euler equation from household i's first-order conditions
2) Use market clearing condition (MCC) for each good
3) Find q

Problem 2

(c) Assume $\sigma = 1$ and assume β can be any number between 0 and 1. What is equilibrium price q?

1) Find consumption for both goods and consumers
2) Use MCC for each good
3) Find q
4) Check signs of $\frac{\partial q^*}{\partial \beta}, \frac{\partial c_1^{i*}}{\partial \beta}, \frac{\partial c_2^{i*}}{\partial \beta}$ for $i = 1, 2$

Problem 3

Market and Home Production

(a) Define tax-distorted competitive equilbrium (TDCE)

1) Consumer allocation solves their optimization problem
2) Firm's allocation solves its optimization problem
3) Government budget is balanced
4) Markets clear (market good and labor)

- Think carefully on how to include production of home good

Problem 3

(b) Find the ratio of H_m and H_h in terms of parameters and taxes rates

1) FOC with respect to choice variables
2) $\frac{\partial\left(\frac{H_m}{H_h}\right)}{\partial\left(\frac{A_h}{A_m}\right)}$

(c) Find leisure L in terms of H_h, parameters and tax rate. What happens to leisure as H_h falls ?

1) $H_h + H_m + L = 1$
2) Direct and indirect effects

(d) Suppose you want to comment on the effect on improvement in home appliances on labor supply and leisure. How would you use this model to make that comment?

Problem 4

Cross-Country Differences in Taxes & Labor Supply (Prescott 2004)

(a) Define TDCE

1) Consumer allocation solves their optimization problem
2) Firm allocation solves its optimization problem
3) Government budget is balanced
4) Markets clear

Problem 4

(b) Choose ϕ s.t. at U.S. tax rate, model matches U.S. labor supply

1) FOC with respect to choice variables
2) Use 2 FOCs to find expression that has c, h, w, ϕ, and τ
3) Find ϕ (*Hint:* write wage in terms of y and n; find c/y)

(c) Suppose U.S. adopts tax rates of other countries. Fill table with new levels of labor supply.

1) Using the same expression as point 2) in (b) but now leave h
2) You will get h and ϕ and τ
3) Find h for each τ using ϕ

Problem 5

Useful **Constant Returns to Scale** (CRS) Production Functions

- Cobb-Douglas

$$y_t = f(k_t, n_t; A, \alpha) = A k_t^{\alpha} n_t^{1-\alpha}$$

  - $A > 0$: Total factor productivity
  - $\alpha \in [0, 1]$: Output elasticity of capital

- Constant Elasticity of Subsitition (CES)

$$y_t = f(k_t, n_t; \sigma, \alpha) = \left(\alpha^{\frac{1}{\sigma}} k_t^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha)^{\frac{1}{\sigma}} n_t^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}}$$

  - $\sigma \in (0, \infty)$: Elasticity of Substitution Capital-Labor
  - $\alpha \in [0, 1]$: Weight of Capital in Production

Problem 5

Assume representative firm has Cobb-Douglas technology

(a) Show wage w_t, rental rate r_t depends on capital per worker k_t/n_t

- Write profit-maximization problem of *price-taking* firm
- Use firm's FOCs for input demand to characterize equilibrium prices

(b) Write wage in terms of y_t/n_t and rental rate in terms of y_t/k_t

- Use "multiply by one trick" on answers to (a)
- E.g., Multiply r_t by k_t/k_t

Problem 5

Assume representative firm has CES technology

(c) Repeat (a) and (b) w/ CES technology

- Be careful w/ chain rule! May help to define $Z_t \equiv \alpha^{\frac{1}{\sigma}} k_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}}$, so $y_t = Z_t^{\frac{\sigma}{\sigma-1}}$
- Easier to first write r_t in terms of y_t/k_t, w_t in terms of y_t/n_t
- Then show $y_t = f(k_t, n_t) = n_t \times f(k_t/n_t, 1)$ and use this fact to finish
- Sanity check: if $\sigma = 1$, CES identical to Cobb-Douglas

(d) What happens to input demand as $\sigma \to 0$ and $\sigma \to \infty$?

- σ determines whether capital-labor are substitutes or complements

Useful Properties of CRS Technology

- Simplify production sector w/ single **representative firm**
  - Total output depends only on total inputs and *not* the size distribution of firms
  - **Exercise:** Assume economy has J firms. Denote each firm's share of aggregate capital K and aggregate labor N as s_j. Show that

$$\sum_{j=1}^{J} f(s_j K, s_j N) = f(K, N)$$

    for Cobb-Douglas, CES, and any CRS production function $f(K, N)$

Useful Properties of CRS Technology (cont.)

- Zero-profit condition (under *competitive* input and output markets)
  - W/ CRS technology and perfect competition, firm's total revenue paid *entirely* to production factors
  - Zero profits in equilibrium simplifies national income accounting
  - **Exercise:** Using your expressions for r_t, w_t and profit equation, show that representative firm makes zero profit in equilibrium
- Factor prices depend only on *ratio* of inputs K/N, not levels K or N
  - Allows for normalization of total inputs w/out loss of generality
  - Common example: Normalize total labor force $N = 1$
  - Helpful for finding steady-state allocations, prices in growth models
  - **You show this in Problem 5**

Computation Exercise 1

Due Tuesday, Sept. 9 at 2:00p.m.

- Three exercises to be coded in Matlab
- You will submit
  1. PDF with answers (math, plots, tables, etc.)
  2. Matlab scripts that reproduce results shown in the PDF
     - You can upload a single Matlab file, or `.zip` of multiple scripts
     - When grading, I expect scripts to run fully with only minor changes

Computation Exercise 1 – Preview

$$u(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} & \text{for } \sigma \neq 1 \\ \log(c) & \text{for } \sigma = 1 \end{cases}$$

1. Plot CRRA utility, marginal utility for different σ values
   - $\sigma \geq 0$ is coefficient of relative risk aversion
   - Satisfies **Inada conditions**, which preclude corner solutions
     - $\lim_{c \to 0} u'(c) = \infty$
     - $\lim_{c \to \infty} u'(c) = 0$
   - **Homothetic:** income does not affect ratio of goods consumed
     - Useful for aggregation (see MWG Proposition 4.C.2)
   - Distinct class of utility functions consistent w/ **balanced growth path**

Computation Exercise 1 – Preview

2. Modeling ag, manufacturing, and services share of expenditure

$$\max_{c_a, c_m, c_s} \omega_a \log(c_a + \bar{c}_a) + \omega_m \log(c_m + \bar{c}_m) + \omega_s \log(c_s + \bar{c}_s)$$

subject to

$$\sum_{i \in \{a,m,s\}} p_i c_i = C$$

a) Find expenditure shares $s_i \equiv \frac{p_i c_i}{C}$ for $i \in \{a, m, s\}$
   - Write three first-order conditions
   - Rewrite budget constraint in terms of only one unknown to solve

b) Evaluate sign of $\frac{\partial s_i}{\partial C}$ for $i \in \{a, m, s\}$, assuming $\bar{c}_a < 0$, $\bar{c}_m = 0$, $\bar{c}_s > 0$

Computation Exercise 1 – Preview

2. Modeling ag, manufacturing, and services share of expenditure
    - c) Plot U.S. data for ag, manufacturing, and services shares
    - d) "Calibrate" model parameters $(\bar{c}_a, \bar{c}_s)$ to fit "targeted moments"
    - e) Interpret values of $\bar{c}_a$ and $\bar{c}_s$
    - f) Plot expenditure shares simulated from model against data
3. Newton's Method
    - Numerical technique for solving systems of equations
    - To be used in future computation exercises