

TA Session 3

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ECON 8040: Macroeconomics I

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Announcements

Midterm Exam

Thursday, October 16, 2:20 p.m. – 3:35 p.m.

MLC 275

- ▶ Problem Set 0 grades posted to eLC
 - Email me if you have any questions about grading

Today's Session

- ▶ Problem Set 0 Review
 - Not all graded yet due to an extension
 - I will discuss grades only in broad terms today
 - Detailed solutions later
- ▶ Problem Set 1 due **Friday, Sept. 5 at 11:59 p.m.**
- ▶ Computation Exercise 1 due **Tuesday, Sept. 9 at 2:00 p.m.**

Grading Comments

- ▶ General rules for good data visualization
 - **Time series** displayed as *lines*
 - **Cross-section** displayed as *scatter plot*
 - Titles, labels should be **human-readable words**, not machine-readable variables
 - Try your best to align plots and text near one another in \LaTeX

Remarks on #9

- ▶ Keep CE definition separate from work of solving it
 - Don't include extraneous variables (e.g., no capital in this problem)
 - Don't write FOCs in CE definition in part (a)
 - Don't write answers in part (b) w/out showing some work
- ▶ Broad structure of CE definition
 - “CE is prices, policy, and allocations such that ...”
 - Given prices and policy, HH allocation maximizes preferences
 - Given prices and policy, firm maximizes profit
 - Government (if applicable)
 - Markets clear

Problem Set 1

Due Friday, Sept. 5 at 11:59 p.m.

- ▶ Five static models that you will solve analytically
- ▶ Write verbose competitive equilibrium definitions
 - State all equilibrium prices and allocations *first*
 - State clearly which equilibrium objects households / firms *take as given* and what they *choose*
 - Refer to previous slide + TA2 slides on PS0 #9 for an example
- ▶ Generally ordered easiest to hardest
- ▶ If stuck, move on to the next problem
- ▶ If provided hints don't help, email me for additional hints

Problem 1

Economy w/ Indivisible Labor Supply

- (a) Define competitive equilibrium
- (b) In equilibrium, households are *indifferent* between working full-time and not working at all
 - 1 Use this condition to write down an equation
 - Think carefully about how much workers/non-workers consume
 - 2 Solve for $\frac{r^*}{w^*}$ (it equals a constant)
 - 3 Write down firm's FOCs
 - 4 Combine expression from steps 2 and 3 to write an equation that has equilibrium labor supply n^* as its only variable and solve

Problem 2

Two-Good Economy

(a) Define competitive equilibrium.

- 1) Consumer 1's allocation solves their optimization problem
- 2) Consumer 2's allocation solves their optimization problem
- 3) Markets clear

(b) Suppose $\beta = 0.5$. What is equilibrium price \mathbf{q} ?

- 1) Write Euler equation from household i 's first-order conditions
- 2) Use market clearing condition (MCC) for each good
- 3) Find \mathbf{q}

Problem 2

(c) Assume $\sigma = 1$ and assume β can be any number between 0 and 1. What is equilibrium price q ?

- 1) Find consumption for both goods and consumers
- 2) Use MCC for each good
- 3) Find q
- 4) Check signs of $\frac{\partial q^*}{\partial \beta}$, $\frac{\partial c_1^*}{\partial \beta}$, $\frac{\partial c_2^*}{\partial \beta}$ for $i = 1, 2$

Problem 3

Market and Home Production

- (a)** Define tax-distorted competitive equilibrium (TDCE)
- 1) Consumer allocation solves their optimization problem
 - 2) Firm's allocation solves its optimization problem
 - 3) Government budget is balanced
 - 4) Markets clear (market good and labor)
- Think carefully on how to include production of home good

Problem 3

- (b)** Find the ratio of H_m and H_h in terms of parameters and taxes rates
- 1) FOC with respect to choice variables
 - 2) $\frac{\partial \left(\frac{H_m}{H_h} \right)}{\partial \left(\frac{A_h}{A_m} \right)}$
- (c)** Find leisure L in terms of H_h , parameters and tax rate. What happens to leisure as H_h falls ?
- 1) $H_h + H_m + L = 1$
 - 2) Direct and indirect effects
- (d)** Suppose you want to comment on the effect on improvement in home appliances on labor supply and leisure. How would you use this model to make that comment?

Problem 4

Cross-Country Differences in Taxes & Labor Supply ([Prescott 2004](#))

(a) Define TDCE

- 1) Consumer allocation solves their optimization problem
- 2) Firm allocation solves its optimization problem
- 3) Government budget is balanced
- 4) Markets clear

Problem 4

- (b)** Choose ϕ s.t. at U.S. tax rate, model matches U.S. labor supply
- 1) FOC with respect to choice variables
 - 2) Use 2 FOCs to find expression that has \mathbf{c} , \mathbf{h} , \mathbf{w} , ϕ , and τ
 - 3) Find ϕ (*Hint: write wage in terms of \mathbf{y} and \mathbf{n} ; find \mathbf{c}/\mathbf{y}*)
- (c)** Suppose U.S. adopts tax rates of other countries. Fill table with new levels of labor supply.
- 1) Using the same expression as point 2) in (b) but now leave \mathbf{h}
 - 2) You will get \mathbf{h} and ϕ and τ
 - 3) Find \mathbf{h} for each τ using ϕ

Problem 5

Useful **Constant Returns to Scale** (CRS) Production Functions

► Cobb-Douglas

$$y_t = f(k_t, n_t; A, \alpha) = Ak_t^\alpha n_t^{1-\alpha}$$

- $A > 0$: Total factor productivity
- $\alpha \in [0, 1]$: Output elasticity of capital

► Constant Elasticity of Substitution (CES)

$$y_t = f(k_t, n_t; \sigma, \alpha) = \left(\alpha^{\frac{1}{\sigma}} k_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

- $\sigma \in (0, \infty)$: Elasticity of Substitution Capital-Labor
- $\alpha \in [0, 1]$: Weight of Capital in Production

Problem 5

Assume representative firm has Cobb-Douglas technology

- (a)** Show wage w_t , rental rate r_t depends on capital per worker k_t/n_t
- Write profit-maximization problem of *price-taking* firm
 - Use firm's FOCs for input demand to characterize equilibrium prices
- (b)** Write wage in terms of y_t/n_t and rental rate in terms of y_t/k_t
- Use “multiply by one trick” on answers to (a)
 - E.g., Multiply r_t by k_t/k_t

Problem 5

Assume representative firm has CES technology

(c) Repeat (a) and (b) w/ CES technology

- Be careful w/ chain rule! May help to define

$$Z_t \equiv \alpha^{\frac{1}{\sigma}} k_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)^{\frac{1}{\sigma}} n_t^{\frac{\sigma-1}{\sigma}}, \text{ so } y_t = Z_t^{\frac{\sigma}{\sigma-1}}$$

- Easier to first write r_t in terms of y_t/k_t , w_t in terms of y_t/n_t
- Then show $y_t = f(k_t, n_t) = n_t \times f(k_t/n_t, 1)$ and use this fact to finish
- Sanity check: if $\sigma = 1$, CES identical to Cobb-Douglas

(d) What happens to input demand as $\sigma \rightarrow 0$ and $\sigma \rightarrow \infty$?

- σ determines whether capital-labor are substitutes or complements

Useful Properties of CRS Technology

- ▶ Simplify production sector w/ single **representative firm**
 - Total output depends only on total inputs and *not* the size distribution of firms
 - **Exercise:** Assume economy has J firms. Denote each firm's share of aggregate capital K and aggregate labor N as s_j . Show that

$$\sum_{j=1}^J f(s_j K, s_j N) = f(K, N)$$

for Cobb-Douglas, CES, and any CRS production function $f(K, N)$

Useful Properties of CRS Technology (cont.)

- ▶ Zero-profit condition (under *competitive* input and output markets)
 - W/ CRS technology and perfect competition, firm's total revenue paid *entirely* to production factors
 - Zero profits in equilibrium simplifies national income accounting
 - **Exercise:** Using your expressions for r_t , w_t and profit equation, show that representative firm makes zero profit in equilibrium
- ▶ Factor prices depend only on *ratio* of inputs K/N , not levels K or N
 - Allows for normalization of total inputs w/out loss of generality
 - Common example: Normalize total labor force $N = 1$
 - Helpful for finding steady-state allocations, prices in growth models
 - **You show this in Problem 5**

Computation Exercise 1

Due Tuesday, Sept. 9 at 2:00p.m.

- ▶ Three exercises to be coded in Matlab
- ▶ You will submit
 - 1 PDF with answers (math, plots, tables, etc.)
 - 2 Matlab scripts that reproduce results shown in the PDF
 - You can upload a single Matlab file, or .zip of multiple scripts
 - When grading, I expect scripts to run fully with only minor changes

Computation Exercise 1 – Preview

$$u(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} & \text{for } \sigma \neq 1 \\ \log(c) & \text{for } \sigma = 1 \end{cases}$$

- 1 Plot CRRA utility, marginal utility for different σ values
 - $\sigma \geq 0$ is coefficient of relative risk aversion
 - Satisfies **Inada conditions**, which preclude corner solutions
 - $\lim_{c \rightarrow 0} u'(c) = \infty$
 - $\lim_{c \rightarrow \infty} u'(c) = 0$
 - **Homothetic**: income does not affect ratio of goods consumed
 - Useful for aggregation (see MWG Proposition 4.C.2)
 - Distinct class of utility functions consistent w/ **balanced growth path**

Computation Exercise 1 – Preview

- ② Modeling ag, manufacturing, and services share of expenditure

$$\max_{c_a, c_m, c_s} \omega_a \log(c_a + \bar{c}_a) + \omega_m \log(c_m + \bar{c}_m) + \omega_s \log(c_s + \bar{c}_s)$$

subject to

$$\sum_{i \in \{a, m, s\}} p_i c_i = C$$

- a) Find expenditure shares $s_i \equiv \frac{p_i c_i}{C}$ for $i \in \{a, m, s\}$
- Write three first-order conditions
 - Rewrite budget constraint in terms of only one unknown to solve
- b) Evaluate sign of $\frac{\partial s_i}{\partial C}$ for $i \in \{a, m, s\}$, assuming $\bar{c}_a < 0$, $\bar{c}_m = 0$, $\bar{c}_s > 0$

Computation Exercise 1 – Preview

- 2 Modeling ag, manufacturing, and services share of expenditure
 - c) Plot U.S. data for ag, manufacturing, and services shares
 - d) “Calibrate” model parameters (\bar{c}_a, \bar{c}_s) to fit “targeted moments”
 - e) Interpret values of \bar{c}_a and \bar{c}_s
 - f) Plot expenditure shares simulated from model against data
- 3 Newton’s Method
 - Numerical technique for solving systems of equations
 - To be used in future computation exercises

About Matlab

- ▶ Popular programming language among economists
- ▶ Advantages:
 - Efficient matrix math
 - Built-in functionality
 - Simple syntax
 - Documentation
- ▶ Disadvantages:
 - Proprietary
 - Limited add-ons
 - Non-numeric data

Getting Started

- ▶ [UGA installation guide](#)
- ▶ Useful add-ons / toolboxes
 - Optimization
 - Parallel Computing
 - Econometrics
- ▶ Resources available on eLC under Content > Matlab
 - [Matlab documentation](#)
 - Tutorial Scripts (.zip)
 - Guides (links and PDFs)