

# Final Exam Review Session

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ECON 8040: Macroeconomics I

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# Today's Session

- ▶ **Problem Set 7** grades on eLC
- ▶ **Computation Exercise 2** grades on eLC
- ▶ **Final Exam**, Thursday, December 4, 3:30–6:30 p.m., MLC 275

## Problem Set 7

- ▶ Recursive planner's problems (Problems 1 and 2)
- ▶ Asset pricing (Problems 3 and 5)
- ▶ Infinite horizon economy w/ production (Problem 4)

# Problem 1

Infinite-horizon production economy with household and business sector.

(a) Write down social planner's problem

Replacing  $n_{Ht} = 1 - N_{Mt}$  and  $c_{Ht} = k_{Ht}^\alpha n_{Ht}^{1-\alpha}$ , the planner's problem is

$$\max_{\{c_{Mt}, N_{Mt}, K_{Mt+1}, k_{Ht+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log \left( \left[ \mu c_{Mt}^\rho + (1 - \mu) [k_{Ht}^\alpha (1 - N_{Mt})^{1-\alpha}]^\rho \right]^{\frac{1}{\rho}} \right)$$

subject to

$$c_{Mt} + K_{Mt+1} + k_{Ht+1} = K_{Mt}^\alpha N_{Mt}^{1-\alpha} + (1 - \delta_K) K_{Mt} + (1 - \delta_H) k_{Ht}$$

$$c_{Mt}, K_{Mt+1}, k_{Ht+1} \geq 0$$

$$0 \leq N_{Mt} \leq 1$$

$$K_{M0}, k_{H0} \text{ given}$$

## Problem 1 (cont.)

(b) Write it recursively

$$V(K_M, k_H) = \max_{\{c_M, N_M, K'_M, k'_H\}} \left\{ \log \left( \left[ \mu c_M^\rho + (1 - \mu) [k_H^\alpha (1 - N_M)^{1-\alpha}]^\rho \right]^{\frac{1}{\rho}} \right) + \beta V(K'_M, k'_H) \right\}$$

subject to

$$\begin{aligned} c_M &= K_M^\alpha N_M^{1-\alpha} + (1 - \delta_K) K_M + (1 - \delta_H) k_H - K'_M - k'_H; \lambda \\ 0 &\leq K'_M, k'_H, K'_M + k'_H \leq K_M^\alpha N_M^{1-\alpha} + (1 - \delta_K) K_M + (1 - \delta_H) k_H \\ 0 &\leq N_M \leq 1 \\ K_{M0}, k_{H0} &\text{ given} \end{aligned}$$

## Problem 1 (cont.)

(c) Write FOC and envelope condition for Bellman

We have four choice variables. The first-order conditions are

$$\text{wrt } c_M: \quad \lambda = \frac{\mu c_M^{\rho-1}}{\mu c_M^\rho + (1-\mu)[k_H^\alpha(1-N_M)^{1-\alpha}]^\rho}$$

$$\text{wrt } N_M: \quad \lambda \left( \frac{K_M}{N_M} \right)^\alpha = \frac{1}{1-N_M} \cdot \frac{(1-\mu)[k_H^\alpha(1-N_M)^{1-\alpha}]^\rho}{\mu c_M^\rho + (1-\mu)[k_H^\alpha(1-N_M)^{1-\alpha}]^\rho}$$

$$\text{wrt } K'_M: \quad \lambda = \beta \frac{\partial V(K'_M, k'_H)}{\partial K'_M}$$

$$\text{wrt } k'_H: \quad \lambda = \beta \frac{\partial V(K'_M, k'_H)}{\partial k'_H}$$

## Problem 1 (cont.)

We have two state variables. The envelope conditions are

$$\text{wrt } K_M: \quad \frac{\partial V(K_M, k_H)}{\partial K_M} = \lambda(\alpha K_M^{\alpha-1} N_M^{1-\alpha} + 1 - \delta_K)$$

$$\begin{aligned} \text{wrt } k_H: \quad \frac{\partial V(K_M, k_H)}{\partial k_H} &= \lambda(1 - \delta_H) + \frac{1}{k_H} \cdot \frac{(1 - \mu)\alpha[k_H^\alpha(1 - N_M)^{1-\alpha}]^\rho}{\mu c_M^\rho + (1 - \mu)[k_H^\alpha(1 - N_M)^{1-\alpha}]^\rho} \\ &= \lambda \left( 1 - \delta_H + \frac{(1 - \mu)\alpha[k_H^\alpha(1 - N_M)^{1-\alpha}]^\rho}{\mu k_H c_M^{\rho-1}} \right) \end{aligned}$$

## Problem 1 (cont.)

(d) Equations that characterize the steady state

We write four equations in four unknowns that can, in principle, be solved

$$\begin{aligned}
 c_M &= K_M^\alpha N_M^{1-\alpha} - \delta_K K_M - \delta_H k_H \\
 \mu c_M^{\rho-1} \left( \frac{K_M}{N_M} \right)^\alpha &= \frac{(1-\mu)[k_H^\alpha (1-N_M)^{1-\alpha}]^\rho}{1-N_M} \\
 \frac{1}{\beta} &= \alpha K_M^{\alpha-1} N_M^{1-\alpha} + 1 - \delta_K \\
 \frac{1}{\beta} &= 1 - \delta_H + \frac{(1-\mu)\alpha [k_H^\alpha (1-N_M)^{1-\alpha}]^\rho}{\mu k_H c_M^{\rho-1}}
 \end{aligned}$$

## Problem 2

### Endogenous mortality model

(a) Write planning problem

$$\max_{\{N_{t+1}, c_t, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} N_t u(c_t)$$

subject to

$$N_t(c_t + ph_t) = N_t y \quad [\text{Feasibility}]$$

$$N_{t+1} = \left(1 - \frac{1}{f(h_t)}\right) N_t \quad [\text{"Law of Motion"}]$$

$$c_t, h_{t+1}, N_{t+1} \geq 0; \quad N_0, y \text{ given}$$

## Problem 2 (cont.)

(b) Recursive planning problem

$$V(N) = \max_{N'} \left\{ \max_{c,h} \{Nu(c)\} + V(N') \right\}$$

subject to

$$N(c + ph) = Ny$$

$$N' = \left(1 - \frac{1}{f(h)}\right) N$$

$$N', c, h \geq 0; \quad y \text{ given}$$

► The Bellman operator does not *discount*

## Problem 2 (cont.)

- (c) Verify  $V(N) = vN$  solves Bellman. Write equation that describes  $v$ .
- Solve by procedure akin to guess-and-verify

$$v = u(c)f(h)$$

New Bellman is

$$u(c)f(h)N = \max_{N'} \left\{ \max_{c,h} \{Nu(c)\} + u(c)f(h)N' \right\}$$

subject to  $N' = \left(1 - \frac{1}{f(h)}\right) N; \lambda_1$        $N(c + ph) = Ny; \lambda_2$

## Problem 2 (cont.)

(d) Write down FOCs and characterize solution

$$\text{wrt } c: \quad Nu'(c) + u'(c)f(h)N' = \lambda_2 N$$

$$\text{wrt } h: \quad u(c)f'(h)N' + \frac{\lambda_1 N f'(h)}{f(h)^2} = \lambda_2 N p$$

$$\text{wrt } N': \quad u(c)f(h) = \lambda_1$$

Using FOCs, we find that

$$p = \frac{u(c)f'(h)}{u'(c)f(h)}$$

## Problem 2 (cont.)

(e) Solve for  $\mathbf{s} \equiv \frac{ph}{y}$ .

We cannot derive closed-form solution of  $\mathbf{s}$ , so we write equation that implicitly defines  $\mathbf{s}$  as a function of parameters.

$$0 = \underbrace{by^{\sigma-1} - \frac{\mathbf{s}}{\alpha(1-\mathbf{s})^\sigma} - \frac{1}{(\sigma-1)(1-\mathbf{s})^{\sigma-1}}}_{R(\mathbf{s}, \alpha, b, \sigma, y)}$$

## Problem 2 (cont.)

- (f) How does  $\mathbf{s}$  change in  $\mathbf{y}$ ,  $\mathbf{b}$ ,  $\mathbf{p}$ ,  $\mathbf{A}$ , and  $\alpha$ ? Is health necessity or luxury? By the implicit function theorem, for given parameter  $\theta$ ,

$$\frac{\partial \mathbf{s}}{\partial \theta} = -\frac{\partial \mathbf{s}}{\partial R} \frac{\partial R}{\partial \theta} = -\frac{\partial R / \partial \theta}{\partial R / \partial \mathbf{s}}$$

- $\mathbf{s}$  is unchanging in  $\mathbf{A}$  and  $\mathbf{p}$ .  $\mathbf{s}$  increases as  $\mathbf{y}$ ,  $\mathbf{b}$ , and  $\alpha$  increase if and only if

$$\alpha = \frac{\partial f(\mathbf{h})}{\partial \mathbf{h}} \frac{\mathbf{h}}{f(\mathbf{h})} < \frac{(1 - \mathbf{s})^{1+\sigma}}{1 - (1 - \mathbf{s})\mathbf{s}\sigma}$$

This condition holds if health  $f(\mathbf{h})$  is sufficiently elastic with respect to  $\mathbf{h}$ . Under this condition, health is *luxury*.

## Problem 3

“Lucas Tree” model

(a) SMCE is prices  $\{p_t^b, p_t^s\}_{t=0}^{\infty}$  and household allocation  $\{c_t, b_{t+1}, s_{t+1}\}_{t=0}^{\infty}$  such that

A. Given prices  $\{p_t^b, p_t^s\}_{t=0}^{\infty}$ , household allocation  $\{c_t, b_{t+1}, s_{t+1}\}_{t=0}^{\infty}$  is optimal. That is household allocation solves

$$\max_{\{c_t, b_{t+1}, s_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

## Problem 3 (cont.)

subject to

$$c_t + p_t^b b_{t+1} + p_t^s (s_{t+1} - s_t) \leq b_t + s_t d_t \text{ for } t = 0, 1, \dots,$$

$$c_t, s_{t+1} \geq 0 \text{ for } t = 0, 1, \dots,$$

$$b_{t+1} \geq -\bar{A} \text{ for some } \bar{A} > 0, t = 0, 1, \dots,$$

$$b_0 = 0, s_0 = 1$$

### B. Markets clear

- i. Consumption good:  $c_t = d_t$  for  $t = 0, 1, \dots$  (i.e., all dividends are consumed each period),
- ii. Shares:  $s_t = 1$  for  $t = 0, 1, \dots$  (i.e., all share-holdings among households must sum to 1),
- iii. Assets:  $b_t = 0$  for  $t = 0, 1, \dots$  (i.e., all borrowers have lenders).

## Problem 3 (cont.)

- (b) Recursive Competitive Equilibrium is collection of price functions  $\{p^s(d), p^b(d)\}$ , value function  $\{V^*(w, d)\}$ , and household policy functions  $\{c^*(w, d), b^*(w, d), s^*(w, d)\}$  such that
- A. Given price functions and dividends  $d$  and  $d'$ , household policy functions solve its Bellman equation

$$V(w, d) = \max_{c, b', s'} \{u(c) + \beta V(w', d')\}$$

## Problem 3 (cont.)

subject to

$$c + p^b(d)b' + p^s(d)s' \leq w,$$

$$c, s' \geq 0$$

$$b' \geq -\bar{A} \text{ for some } \bar{A} > 0$$

The solution to the Bellman equation is value function  $V^*(w, d)$ .

B. Price functions are functions of dividends  $d$ .

$$p^s = p^s(d)$$

$$p^b = p^b(d)$$

## Problem 3 (cont.)

- C. Allocation is feasible. That is, household policies satisfy market clearing conditions in aggregate. For all dividends  $d$ ,

$$c(d, d) = d$$

$$b(d, d) = 0$$

$$s(d, d) = 1$$

## Problem 3 (cont.)

(c) Assume  $d_t = 1$  and solve  $p_t^s$  and  $p_t^b$ .

- ▶ Use household's first-order conditions and market-clearing condition to solve bond price

$$p_t^b = \beta$$

- ▶ Write asset pricing equation in terms of future price and dividend. Then use “no bubble” assumption to solve:

$$p_t^s = \frac{\beta}{1 - \beta}$$

## Problem 3 (cont.)

(d) Assume  $d_t = 1$  if  $t$  is even and  $d_t = 2$  if  $t$  is odd.

- ▶ Write down FOCs like in (c). Use dividend growth in even/odd periods to get bond prices:

$$p_t^b = \begin{cases} \frac{\beta}{2} & t = 0, 2, 4, \dots \\ 2\beta & t = 1, 3, 5, \dots \end{cases}$$

## Problem 3 (cont.)

- ▶ Write asset pricing equation and use fact that  $p_t^s = p_{t+2}^s$  and  $d_t = d_{t+2}$  to solve for even/odd period asset prices

$$p_t^s = \begin{cases} \frac{\beta}{1-\beta} & t = 0, 2, 4, \dots \\ \frac{2\beta}{1-\beta} & t = 1, 3, 5, \dots \end{cases}$$

## Problem 4

(Final Exam, Fall 2017) Model with Population Growth and Labor-Augmenting Technology Growth.

This model is consistent with the stylized “Kaldor” facts about growth:

- 1 GDP per worker and capital per worker have grown at sustained rate
- 2 Wages grow at sustained rate
- 3 Real interest rate and capital-output ratio are fairly constant over time
- 4 Factor shares of income are stable

## Problem 4 (cont.)

(a) Aggregate feasibility

$$C_t + K_{t+1} = AK_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t$$

where  $L_t \equiv (1 + \gamma)^t (1 + \eta)^t h_t$  is aggregate effective hours worked.

(b) Write down firm's maximization problem and its FOCs.

$$r_t = (1 - \tau)\alpha A \left(\frac{K_t}{L_t}\right)^{\alpha-1} \quad w_t = (1 - \tau)(1 - \alpha)(1 + \gamma)^t A \left(\frac{K_t}{L_t}\right)^\alpha$$

(c) Nat'l Income =  $(1 - \tau)Y_t = \underbrace{w_t(1 + \eta)^t h_t}_{\text{Labor Income}} + \underbrace{r_t K_t}_{\text{Capital Income}}$

## Problem 4 (cont.)

- (d) SMCE is policy  $\{\tau, Tr_t\}_{t=0}^{\infty}$ , prices  $\{r_t, w_t\}_{t=0}^{\infty}$ , interest rate  $\{i_t\}_{t=0}^{\infty}$ , household allocation  $\{c_t, x_t, k_{t+1}, a_{t+1}, h_t\}_{t=0}^{\infty}$ , and firm allocation  $\{Y_t, K_t^d, h_t^d\}_{t=0}^{\infty}$  such that
- A. Given prices, interest rate, and policy, HH allocation solves

$$\max_{\{c_t, x_t, k_{t+1}, a_{t+1}, h_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (1 + \eta)^t (\phi \log(c_t) + (1 - \phi) \log(1 - h_t))$$

$$\text{subject to } c_t + x_t + (1 + \eta) \frac{a_{t+1}}{1 + i_{t+1}} = w_t h_t + r_t k_t + a_t + Tr_t \text{ for } t = 0, 1, \dots;$$

$$x_t = (1 + \eta) k_{t+1} - (1 - \delta) k_t \text{ for } t = 0, 1, \dots;$$

$$(1 + \eta) a_{t+1} \geq -\bar{A} (1 + \gamma)^t \text{ for some } \bar{A} > 0 \text{ for } t = 0, 1, \dots;$$

$$k_0 \text{ given; } c_t, x_t, k_{t+1} \geq 0 \text{ for } t = 0, 1, \dots; 0 \leq h_t \leq 1 \text{ for } t = 0, 1, \dots; a_0 = 0$$

## Problem 4 (cont.)

- B. Given prices, interest rate, and policy, firm maximizes after-tax profit

$$\max_{\{Y_t, K_t^d, h_t^d\}_{t=0}^{\infty}} (1 - \tau)Y_t - w_t(1 + \eta)^t h_t^d - r_t K_t^d$$

subject to  $Y_t = A(K_t^d)^\alpha [(1 + \gamma)^t (1 + \eta)^t h_t^d]^{1-\alpha}$ .

- C. Government budget balances:  $\tau Y_t = (1 + \eta)^t T r_t$  for  $t = 0, 1, \dots$
- D. Markets clear. In all periods  $t = 0, 1, 2, \dots$ ,
- Consumption good:  $C_t + K_{t+1} = Y_t + (1 - \delta)K_t$
  - Labor:  $(1 + \eta)^t h_t^d = (1 + \eta)^t h_t$
  - Capital:  $K_t^d = (1 + \eta)^t k_t$
  - Asset:  $(1 + \eta)^t a_t = 0$

## Problem 4 (cont.)

- (e) Write three equations in per person variables that describe equilibrium allocations

$$\text{MRS:} \quad w_t = \frac{(1 - \phi)c_t}{\phi(1 - h_t)}$$

$$\text{Euler:} \quad \frac{c_{t+1}}{c_t} = \beta(r_{t+1} + 1 - \delta)$$

$$\text{Feasibility:} \quad c_t + (1 + \eta)k_{t+1} = Ak_t^\alpha [(1 + \gamma)^t h_t]^{1-\alpha} + (1 - \delta)k_t$$

- (f) Find interest rate  $i_t$  in terms of allocations and tax rate.

$$i_{t+1} = (1 - \tau)\alpha A \left( \frac{k_{t+1}}{(1 + \gamma)^{t+1} h_{t+1}} \right)^{\alpha-1} - \delta$$

## Problem 4 (cont.)

- (g) Write aggregate feasibility in terms of de-trended variables, denoted by  $\hat{\cdot}$ . What is long-run growth rate of per-capita variables?

$$\hat{c}_t + (1 + \gamma)(1 + \eta)\hat{k}_{t+1} = A\hat{k}_t^\alpha h_t^{1-\alpha} + (1 - \delta)\hat{k}_t$$

Per-capita output, consumption, capital, and investment all grow at gross rate  $1 + \gamma$ . Hours per capita are constant.

## Problem 4 (cont.)

(h) Write 3 equations that describe long-run (i.e., steady state) values of the de-trended variables:

$$\text{MRS:} \quad \hat{w}_t = (1 - \tau)(1 - \alpha)A \left( \frac{\hat{k}}{h} \right)^\alpha = \frac{(1 - \phi)\hat{c}}{\phi(1 - h)}$$

$$\text{Euler:} \quad (1 - \tau)\alpha A \left( \frac{\hat{k}}{h} \right)^{\alpha-1} + 1 - \delta = \frac{1 + \gamma}{\beta}$$

$$\text{Feasibility:} \quad \hat{c} + [(1 + \gamma)(1 + \eta) - 1 + \delta]\hat{k} = A\hat{k}^\alpha h^{1-\alpha}$$

## Problem 4 (cont.)

- (i) What is long-run effect of  $\tau$  on interest rate?

$$1 + i^* = \frac{1 + \gamma}{\beta}$$

Tax rate  $\tau$  does *not* affect long-run interest rate!

## Problem 4 (cont.)

(j) What is long-run effect of  $\tau$  on  $\frac{\hat{k}}{h}$ ?

$$\frac{\hat{k}}{h} = \left( \frac{(1-\tau)\alpha A}{\frac{1+\gamma}{\beta} - 1 + \delta} \right)^{\frac{1}{1-\alpha}}$$

Long-run capital-labor ratio is *decreasing* in tax rate  $\tau$ .

## Problem 4 (cont.)

(k) Use national accounting data to calibrate model parameters:

$$\delta = 0.02$$

$$\alpha = 0.33$$

$$\phi = 0.36$$

$$\beta = 0.97$$

$$\tau = 0.1$$

## Problem 5

Economy with workers and capitalists (i.e., firm owners)

(a) Define ADCE.

- Worker (type 1) budget constraint:  $c_t^1 = (1 - \tau_L)w_t n_t^1 \forall t$
- Given prices and policy, firm maxes present value of after-tax dividends:

$$\max_{\{d_t, K_{t+1}, N_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1 - \tau_d) p_t d_t$$

subject to  $d_t = K_t^\alpha N_t^{1-\alpha} - w_t N_t - (K_{t+1} - (1 - \delta)K_t) \forall t$

- Government budget balances:  $\tau_L w_t n_t^1 + \tau_d d_t = g K_t^\alpha N_t^{1-\alpha}$
- Market clearing conditions
  - i. Final good:  $c_t^1 + c_t^2 + K_{t+1} = (1 - g)K_t^\alpha N_t^{1-\alpha} + (1 - \delta)K_t$
  - ii. Labor:  $n_t^1 = N_t$
  - iii. Stock:  $s_t = 1$

## Problem 5 (cont.)

- (b) Solve for labor supply of workers; consumption of workers and capitalists.
- ▶ Use worker's FOC and budget constraint (which holds with equality in every period), to find labor supply.

$$n_t^1 = 1/2$$

- ▶ Workers and capitalists consume post-tax income each period

$$c_t^1 = 1/2(1 - \tau_L)w_t$$

$$c_t^2 = (1 - \tau_d)d_t$$

## Problem 5 (cont.)

(c) Write asset pricing equation

$$q_t = \beta \frac{c_t^2}{c_{t+1}^2} (q_{t+1} + (1 - \tau_d)d_{t+1})$$

(d) Firm's first-order conditions

$$w_t = (1 - \alpha)K_t^\alpha N_t^{-\alpha}$$

$$p_t = p_{t+1}(\alpha K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + 1 - \delta)$$

## Problem 5 (cont.)

- (e) Steady state capital stock. Use firm's and capitalist's FOCs (and steady state labor supply  $N$ ):

$$K = \frac{1}{2} \left( \frac{\alpha}{\frac{1}{\beta} - 1 + \delta} \right)^{\frac{1}{1-\alpha}}$$

- (f) Find steady state wage. Use  $K$  and  $N$  in steady state,

$$w = (1 - \alpha) \left( \frac{\alpha}{\frac{1}{\beta} - 1 + \delta} \right)^{\frac{\alpha}{1-\alpha}}$$

## Problem 5 (cont.)

(g) Find steady state share price  $q$ . Start with equation from (c):

$$q = \frac{\beta}{1 - \beta} (1 - \tau_d) d$$

Simplify using firm's FOC wrt  $K_{t+1}$  and household FOCs wrt  $c_t, c_{t+1}$

$$d = \left( \frac{1}{\beta} - 1 \right) K$$

which implies steady-state asset price is

$$q = (1 - \tau_d) K$$

## Problem 5 (cont.)

- (h) Dividend tax cut increases stock price  $q$  and increases consumption inequality between capitalists and workers. Taxes do not affect other variables (capital stock, wage, labor supply, output, and aggregate consumption)
- (i) Dividend tax cut does not affect output, so it would not “pay for itself.” Government must raise labor income tax to maintain expenditure policy  $gY$ .

## Problem 1

Solve two period model using Newton's method

- (a) Jacobian for the 3 x 3 system (i.e., two budget constraints and one Euler equation):

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -\alpha k_2^{\alpha-1} - (1-\delta) \\ -\sigma c_1^{-\sigma-1} & \sigma c_2^{-\sigma-1} \beta (1-\delta + \alpha k_2^{\alpha-1}) & -(\alpha-1)c_2^{-\sigma} \beta \alpha k_2^{\alpha-2} \end{bmatrix}$$

Numerical solution (within 0.001 margin of error),

$$c_1 \approx 3.7363; c_2 \approx 3.8593; k_2 \approx 2.6478$$

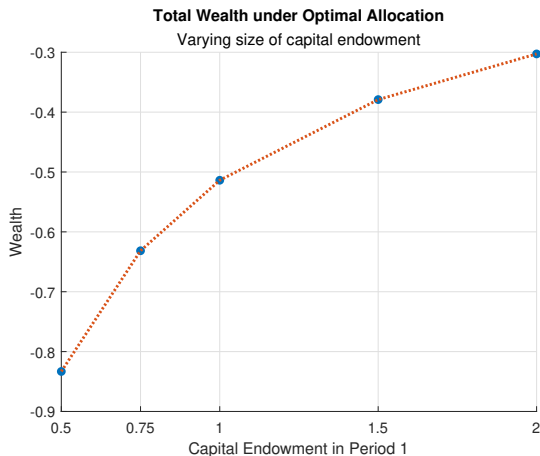
## Problem 1 (cont.)

(b) Solve  $c_1$ ,  $c_2$ ,  $k_2$ , and  $w(\bar{k}_1)$  for different values of  $\bar{k}_1$

Allocation	$\frac{1}{2}k_{ss}$	$\frac{3}{4}k_{ss}$	$k_{ss}$	$\frac{3}{2}k_{ss}$	$2k_{ss}$
$c_1$	2.2576	3.0157	3.7363	5.1142	6.4412
$c_2$	2.4344	3.1673	3.8593	5.1758	6.4391
$k_2$	1.4246	2.0411	2.6478	3.8464	5.0336
$w(\bar{k}_1)$	-0.8332	-0.6315	-0.5138	-0.3791	-0.3028

## Problem 1 (cont.)

(b) Plot  $w(\bar{k}_1)$  for values in  $\mathcal{K} = \{\frac{1}{2}k_{SS}, \frac{3}{4}k_{SS}, k_{SS}, \frac{3}{2}k_{SS}, 2k_{SS}\}$



## Problem 2

(a) Rewrite planner problem so that it's stationary

– Define  $\mathbf{c}_t \equiv \frac{C_t}{(1+\gamma)^t N_t}$  and  $\mathbf{k}_t \equiv \frac{K_t}{(1+\gamma)^t N_t}$

(b) Write problem recursively

$$V(\mathbf{k}_t) = \max_{\mathbf{c}_t, \mathbf{k}_{t+1} \geq 0} \left\{ \frac{\mathbf{c}_t^{1-\sigma}}{1-\sigma} + (1+\gamma)^{1-\sigma} \beta V(\mathbf{k}_{t+1}) \right\}$$

subject to

$$\mathbf{c}_t + (1+\gamma)(1+\eta)\mathbf{k}_{t+1} = A\mathbf{k}_t^\alpha + (1-\delta)\mathbf{k}_t$$

$$\mathbf{c}_t, \mathbf{k}_{t+1} \geq 0, \mathbf{k}_0 \text{ given}$$

## Problem 2 (cont.)

Solve for  $k_{ss}$

- Derive Euler equation from first-order conditions
- **Per effective worker** variables  $c_t$ ,  $k_t$  are constant in steady state

(c) Calibrate parameters of the model. Given  $\sigma$ ,  $\eta$ , and  $\gamma$ ,

①  $\alpha = \frac{r_t k_t}{Y_t} = 1/3$

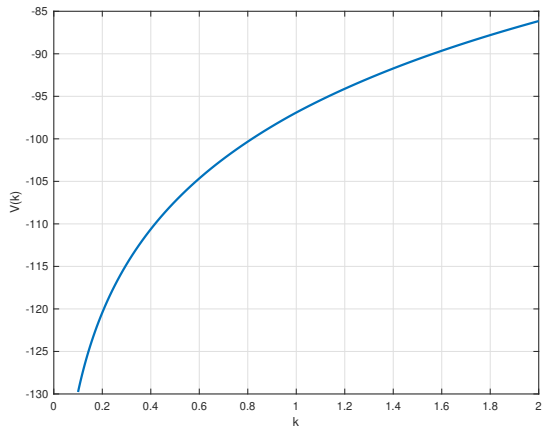
② Use  $\frac{K_{t+1} - (1 - \delta)K_t}{Y_t} = 0.21$  and  $\frac{K_t}{Y_t} = 3$  to solve for  $\delta$

③ Normalize  $k^* = 1$  and use  $\alpha \frac{Y_t}{K_t}$  to solve for  $A$

④ Use Euler equation, other parameters, to solve for  $\beta$

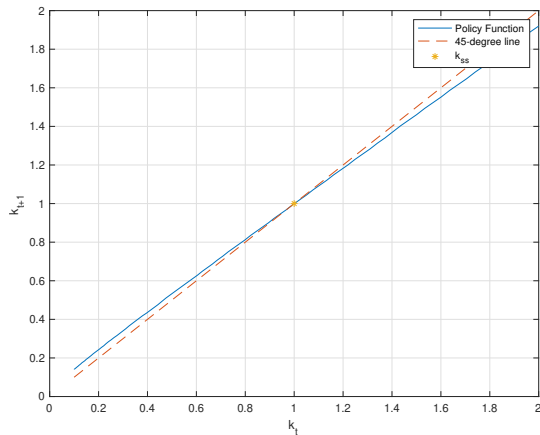
## Problem 2 (cont.)

(d) Plot value function ...



## Problem 2 (cont.)

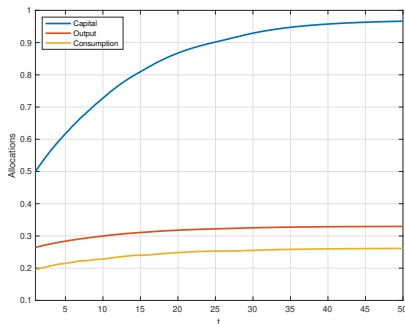
(d) ... and the policy function



## Problem 2 (cont.)

- (e) Set  $k_0 = 0.5k_{ss}$ . Use your approximation of policy function  $g(k)$  from (d) to generate sequence of capital  $\{k_1, \dots, k_{50}\}$ . Then use production function and feasibility to generate sequence of output and consumption.

## Problem 2 (cont.)



- $k_t \rightarrow k^*$
- $\frac{k_t}{y_t} = \frac{K_t}{Y_t} \rightarrow 3$
- In general, good to check that equations used to calibrate the model are satisfied by results in “long-run” (in this case,  $t = 50$ )

## Second-Half Outline

- ▶ Assignments after midterm: PS5, PS6, PS7, CE2
- ▶ Past final exams (2017 – 2024) on eLC
- ▶ Dynamic CE
  - Arrow-Debreu
  - Sequential Markets
  - Negishi Method
- ▶ Growth Models & Steady State
- ▶ Search Models
- ▶ *Note:* See **TA9** for First-half material review