TA Session 13

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ECON 8040: Macroeconomics I
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Today's Session

- **Problem Set 6** grades on eLC
- **Problem Set 7** due **Friday, November 21** at 11:59 p.m.
- **Computation Exercise 2** due **Tuesday, November 25** at 11:59 p.m.
- No TA Session next week
  - University closed for Thanksgiving holiday
- **Final Exam Review Session**
  - Wednesday, December 3, 12:30–2:00, Ivester E007

Problem Set 6

- Four dynamic endowment economies
  - Do the problems in order
  - Problem 1 generalizes ADCE example in lecture notes
  - Problem 2-4 tweak endowments or parameters slightly
  - Process for defining/solving ADCE and SMCE consistent throughout
- Bonus problem (worth 10 points) from previous midterm

Problem 1

(a) ADCE is price $\{p_t\}_{t=0}^{\infty}$ and allocations $\{c_t^1, c_t^2\}_{t=0}^{\infty}$ such that

(A) Given prices, household $i = 1, 2$ makes allocation that solves utility maximization

$$\max_{\{c_t^i\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left(\frac{\left(c_t^i\right)^{1-\sigma} - 1}{1-\sigma} \right)$$

subject to

$$\sum_{t=0}^{\infty} p_t c_t^i = \sum_{t=0}^{\infty} p_t e_t^i$$

$$c_t^i \geq 0 \text{ for all } t$$

(B) Markets clear: $c_t^1 + c_t^2 = 2\gamma^t$ for all t

Problem 1 (cont.)

(b) Household i's Euler equation is:

$$\left(c_t^i\right)^{-\sigma} = \beta \left(\frac{p_t}{p_{t+1}}\right) \left(c_{t+1}^i\right)^{-\sigma}$$

(c) Efficient allocations: Defining $\alpha \equiv \frac{\alpha_2}{\alpha_1}$

$$c_t^1 = \frac{2\gamma^t}{1+\alpha^{1/\sigma}}$$
$$c_t^2 = \frac{2\gamma^t \alpha^{1/\sigma}}{1+\alpha^{1/\sigma}}$$

Problem 1 (cont.)

(d) Negishi method

1. Use "shadow price" λ_t and efficient allocations to find p_t
2. Use p_t and budget constraint to find α s.t. efficient allocation is affordable *without* lump-sum transfer
3. Replace α in efficient allocations from (c)

$$p_t = \left(\beta\gamma^{-\sigma}\right)^t \qquad c_t^1 = \frac{2\gamma^t}{1+\beta\gamma^{1-\sigma}} \qquad c_t^2 = \frac{2\gamma^t\beta\gamma^{1-\sigma}}{1+\beta\gamma^{1-\sigma}}$$

Problem 1 (cont.)

(e) Pareto weights that make equal allocations (γ^t, γ^t) efficient and transfers that result in equal allocations in CE are:

$$\alpha_1 = \alpha_2 = 1/2 \quad t^1\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{-1}{1+\beta\gamma^{1-\sigma}} \quad t^2\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{1+\beta\gamma^{1-\sigma}}$$

Exercise. Define and solve ADCE with these transfers. Verify that allocations $(c_t^{1*}, c_t^{2*}) = (\gamma^t, \gamma^t)$

- Hint 1: Assume there is a government that enforces lump-sum tax/transfer policy. Write government's balancing equation in ADCE
- Hint 2: Transfer is made only once in period 0
- Hint 3: Maintain assumption that $\beta\gamma^{1-\sigma} < 1$

Problem 1 (cont.)

(f) How do consumption growth and price depend on γ, σ?

- Consumption growth increasing in γ, not changing in σ

$$\frac{c_{t+1}^1 + c_{t+1}^2}{c_t^1 + c_t^2} = \frac{2\gamma^{t+1}}{2\gamma^t} = \gamma$$

- This is Walras' Law at work
- Price decreasing in γ

$$\frac{\partial p_t}{\partial \gamma} = \frac{-\beta^t \sigma t}{\gamma^{\sigma t+1}} < 0 \text{ for } t > 0$$

Problem 1 (cont.)

- How price changes in risk-aversion σ is more complicated: sign of the derivative changes with sign on $\log \gamma$

$$\frac{\partial p_t}{\partial \sigma} = \frac{-\beta^t t \log(\gamma)}{\gamma^{\sigma t}}$$
$$\begin{cases} < 0 & \text{for } \gamma > 1, t > 0 \\ = 0 & \text{for } \gamma = 1, t > 0 \\ > 0 & \text{for } \gamma < 1, t > 0 \end{cases}$$

Problem 1 (cont.)

(g) Define SMCE

- Ponzi scheme constraint grows with endowments:

$$a^i_{t+1} \geq -\bar{A}\gamma^t \text{ for some } \bar{A} > 0, \text{ for all } t$$
$$a^i_0 = 0 \text{ implicitly assumed}$$

- Find interest rate. How does it change with γ, σ?

$$1 + i_{t+1} = \frac{\gamma^\sigma}{\beta}$$

- Increasing in γ $$\frac{\partial(1 + i_{t+1})}{\partial\gamma} = \frac{\sigma\gamma^{\sigma-1}}{\beta} > 0$$

Problem 1 (cont.)

- Like Arrow-Debreu price, how interest rates change in σ depends on $\gamma \lesseqgtr 1$

$$\frac{\partial(1+i_{t+1})}{\partial\sigma} = \frac{\gamma^{\sigma}\log(\gamma)}{\beta}$$

$$\begin{cases} >0 & \text{if } \gamma > 1 \\ =0 & \text{if } \gamma = 1 \\ <0 & \text{if } \gamma < 1 \end{cases}$$

Problem 1 (cont.)

- Increase in interest rate has two countervailing effects:
  - **Income effect:** The return on savings is higher, so household consumes more now because they are richer in the future.
  - **Substitution effect:** The return on savings is higher, so household gives up some consumption and increases savings.
- σ controls which effect dominates. With CRRA preferences,
  - "Relative risk aversion" = σ
  - "Intertemporal elasticity of substitution" (IES) = $1/\sigma$
- IES reflects the responsiveness of consumption to changes in the interest rate (or changes in consumption growth):
  - Low IES $\Rightarrow$ "consumption smoothing" (**income effect** dominates)
  - High IES $\Rightarrow$ "consumption tilting" (**substitution effect** dominates)

Problem 1 (cont.)

How does this relate to γ in our model? Suppose $\gamma > 1$:

- Increasing σ decreases IES: "consumption smoothing" dominates
- Endowments are growing, so household needs to *borrow* to bring present consumption closer to future consumption
- Borrowers are the *supply-side* of the Arrow securities market
- Increasing the supply of Arrow security
  - Decreases price of Arrow security
  - Increases interest rate

Through its effect on IES, $\sigma \uparrow \Rightarrow i_{t+1} \uparrow$ if $\gamma > 1$, exactly what our derivative is telling us. Why does increasing σ decrease i_{t+1} when $\gamma < 1$?

Problem 2

(a) Sum of endowments alternate between 4 and 2, so we have to be careful with even and odd periods:

$$p_t = \begin{cases} \beta^t & \text{for } t = 0, 2, 4, \dots \\ 2\beta^t & \text{for } t = 1, 3, 5, \dots \end{cases}$$

$$c_t^1 = \begin{cases} \frac{2}{1+\beta} & \text{for } t = 0, 2, 4, \dots \\ \frac{1}{1+\beta} & \text{for } t = 1, 3, 5, \dots \end{cases}$$

$$c_t^2 = \begin{cases} \frac{2(1+2\beta)}{1+\beta} & \text{for } t = 0, 2, 4, \dots \\ \frac{1+2\beta}{1+\beta} & \text{for } t = 1, 3, 5, \dots \end{cases}$$

Why do prices differ from model in lecture notes?

Problem 2

(b) Define SMCE and find interest rate. Again being mindful of the $t+1$ subscript:

$$1 + i_{t+1} = \begin{cases} \frac{1}{2\beta} & \text{for } t = 0, 2, 4, \ldots \\ \frac{2}{\beta} & \text{for } t = 1, 3, 5, \ldots \end{cases}$$

Problem 3

- No trade in equilbrium.

$$p_t = \begin{cases} \beta^t & \text{for } t = 0, 2, 4, \ldots \\ 2\beta^t & \text{for } t = 1, 3, 5, \ldots \end{cases}$$

Why do prices in this model match prices in Problem 2?

Problem 4

(a) Define ADCE. Similar to #1 except β_i for heterogenous discounting

(b) Efficient allocations: Defining $\alpha \equiv \frac{\alpha_2}{\alpha_1}$ and $\beta \equiv \frac{\beta_2}{\beta_1}$

$$c_t^1 = \frac{2}{1 + \alpha\beta^t} = \frac{2\beta_1^t}{\beta_1^t + \alpha\beta_2^t} \qquad c_t^2 = \frac{2\alpha\beta^t}{1 + \alpha\beta^t} = \frac{2\alpha\beta_2^t}{\beta_1^t + \alpha\beta_2^t}$$

(c) Use Negishi to find CE allocations and Arrow-Debreu price:

$$p_t = \frac{\beta_1^t(1 - \beta_1) + \beta_2^t(1 - \beta_2)}{2 - \beta_1 - \beta_2}$$

$$c_t^1 = \frac{2(1 - \beta_1)\beta_1^t}{\beta_1^t(1 - \beta_1) + \beta_2^t(1 - \beta_2)} \qquad c_t^2 = \frac{2(1 - \beta_2)\beta_2^t}{\beta_1^t(1 - \beta_1) + \beta_2^t(1 - \beta_2)}$$

Problem 4

(d) Household 2 consumes more in the long-run than household 1. By assumption that $\beta_1 < \beta_2$

$$\frac{c_t^1}{c_t^2} = \left(\frac{1-\beta_1}{1-\beta_2}\right)\left(\frac{\beta_1}{\beta_2}\right)^t$$
$$\lim_{t\to\infty}\frac{c_t^1}{c_t^2} = 0$$

(e) Why? Because household 2 is *more patient* than household 1 (Minor) technical point: high discount factor $\Leftrightarrow$ low discount rate

$$\underbrace{\beta}_{\text{“discount factor”}} = \frac{1}{\underbrace{1+\rho}_{\rho\text{ = “discount rate”}}}$$

Problem Set 7

- Due Friday, Nov. 21 at 11:59 p.m.
- Five problems
  - Multi-sector production economy
  - Dynamic model w/ endogenous mortality
  - Asset pricing model
  - Two past exam problems
- This is a long problem set. Don't delay starting it!
- If stuck on one problem, move onto next one (order doesn't matter)

Problem 1

Infinite-horizon production economy w/ business and home sectors

(a) Write down planner's problem
  - Carefully write down all choice variables
  - Output of the business sector can be consumed, or invested in business or home capital
  - Home-produced consumption good cannot be invested

(b) Write the Bellman equation
  - Carefully write down state and control variables
  - Replace constraints in objective where convenient
  - Don't forget to write down all remaining constraints!

Problem 1 (cont.)

(c) Write down FOCs and envelope conditions

- If derivatives are too difficult, adjust Bellman's objective, constraints
- Applying chain rule leads to many cancellations
- To simplify algebra, may be helpful to define

$$Z \equiv \mu c_M^\rho + (1-\mu)[k_H^\alpha (1-N_M)^{1-\alpha}]^\rho$$

(d) Write equations that characterize the steady state

- Feasibility and first-order conditions in steady state
- There should be as many equations as "unknowns." E.g., if you write Bellman w/ four controls, you need to provide four equations in those four unknowns.

Problem 2

Model of endogoneous mortality from Hall and Jones (2007)

(a) Write down planner's problem
   - **Assume discount factor** $\beta = 1$
   - Be careful with choice variables, feasibility and **law of motion**

(b) Write planner's problem recursively. Does it satisfy Blackwell's sufficient conditions?
   - What is the state variable? i.e., what does planner need to know at beginning of period to make optimal allocations?
   - Write down feasibility and law of motion constraints

(c) Show that value function of form *vN* solves the Bellman. What is *v*?

Problem 2 (cont.)

(d) Find p

- Replace v from (c) into Bellman
- Write down and combine FOCs

(e) Use given $u(c)$ and $f(h)$ to find health expenditure share of income

- Find eqn. that **implicitly** defines $s \equiv \frac{ph}{y}$ as function of parameters

$$R(s, \alpha, b, \sigma, y) = 0$$

(f) Comparative statics: how does s change in parameters, p?

- **Implicit function theorem**: For given parameter θ,

$$\frac{\partial s}{\partial \theta} = -\frac{\partial R / \partial \theta}{\partial R / \partial s}$$

Problem 3

The Lucas Tree. (Maybe) a helpful analogy: Imagine a shipwrecked crew on an island. The only consumption good available to crew is fruit of a tree on the island. Each period t tree produces fruit d_t. Crew member with s_t shares of tree is entitled to $s_t d_t$ fruit, and shares are traded at price p_t^s. Crew can also trade one-period risk-free bonds b_t at price p_t^b.

The budget constraint is

$$c_t + p_t^b b_{t+1} + p_t^s (s_{t+1} - s_t) = b_t + s_t d_t$$

Problem 3 (cont.)

(a) Define SMCE. State *all* allocations and market clearing conditions. Write utility as $u(c)$.

(b) Write Recursive Competitive Equilibrium.

- Rewrite budget constraint and use hint that $w \equiv b + s(p^s(d) + d)$
- Allocations in (a) are *policy functions*
- Prices in (a) are functions of dividends
- State variables: (w, d)
- Control variables: (c, s', b')
- Refer to example in lecture notes
- **Hint:** You don't need to do part (b) to solve parts (c) and (d)

Problem 3 (cont.)

- Use SMCE definition in (a) to find p_t^s, p_t^b w/ different dividends d_t

(c) Assume $d_t = 1$

- No utility form provided. Just keep utility as $u(c)$

(d) Assume $u(c) = \log c$ and $d_t = \begin{cases} 1 & t = 0, 2, 4, \ldots \\ 2 & t = 1, 3, 5, \ldots \end{cases}$

- Stock, bond prices each take two values for even/odd periods

Problem 4

Growth model on Fall 2017 Final Exam

- **Advice:** Treat this problem like practice final exam. Set aside your notes and try to do this problem in $\leq$ 3 hours

(a) Write down aggregate feasibility (in terms of aggregate variables (e.g., K_t) or *per person* variables (e.g., $k_t \equiv \frac{K_t}{(1+\eta)^t}$))

(b) Write down firm's profit maximization problem. Derive formulas for rental rate of capital r_t and wage w_t

(c) Write down expression for national income

(d) Define sequential markets competitive equilibrium

Problem 4 (cont.)

(e) Write down three equations using *per person* variables:
   - Euler equation
   - Aggregate feasibility
   - Intratemporal optimality for consumption−leisure

(f) Write equilibrium interest rate i_t in terms of capital-labor ratio, tax and parameters
   - FOC with respect to a_{t+1}

(g) What is the long-run growth rate of *per person* variables? What is the long-run growth rate of hours per person?
   - Don't overthink this one

(g) Detrend all **per person variables** appropriately. Denote de-trended variables with $\widehat{\ }$ (e.g., $\widehat{k}$). Rewrite aggregate feasibility with hats.

Problem 4 (cont.)

(h) Write down three equations in steady state:

1. Euler equation
2. Aggregate feasibility
3. Marginal rate of substitution between consumption and leisure

   – Why are $\widehat{k}$ and other variables constant in the long-run?

- Desribe the long-run effect of τ on
  - (i) interest rate i_t
  - (j) capital per worker $\widehat{k}/h$

(k) Calibrate the parameters $(\delta, \alpha, \phi, \beta, \tau)$ model using provided table

   – Recommended order: τ, α, ϕ, δ, β

Problem 5

Macro Preliminary Exam, Summer 2018

(a) Define Arrow-Debreu competitive equilibrium.
   - Write down all prices and allocations
   - Be careful writing budget constraints for workers and capitalists
   - Don't forget policy variables and government budget equation

(b) Solve for n_t^1
   - Take first order conditions for workers
   - Then use budget constraint

(b) Write c_t^1, c_t^2 in terms of post-tax labor income and post-tax dividend

Problem 5 (cont.)

(c) Solve for q_t in terms of q_{t+1}, d_{t+1}, and consumption growth
   - FOC with respect to s_{t+1}

(d) Firm's first-order conditions with respect to choice variables

(e,f,g) Find steady state capital, wage, and stock price q_t

(h,i) Comment on effects of tax reform

Computation Exercise 2

- Due Tuesday, Nov. 25 at 11:59 p.m.
- Solve finite and infinite horizon models numerically
- Submit Matlab code that **runs**
  - Functions defined at bottom of script or in separate file
  - You should be able to open your m file and click “Run” w/out error
  - Put all plots and tables asked for in your PDF

Problem 1

Two-period planning problem

(a) Use **Newton's Method** to find (c_1, c_2, k_2) when $\bar{k}_1 = k_{ss}$
   - Calculate k_{ss} using using provided parameter values
   - Rewrite three equations s.t. all equal 0
   - Write 3×3 Jacobian matrix
   - Refer to Newton's Method code you wrote for CE1

(b) Find allocations, welfare for other initial capital stocks
   - Do it for all $\mathcal{K} = \{0.5k_{ss}, 0.75k_{ss}, k_{ss}, 1.5k_{ss}, 2k_{ss}\}$
   - Plug c_1, c_2 in utility function to find $w(\bar{k}_1)$

Problem 2

Infinite-horizon planning problem w/ pop. and productivity growth

(a) Rewrite planner's problem so feasibility constraint is stationary
   - Divide aggregates by growth rates to get **per effective worker allocations**
   - **Caution:** Per person consumption in utility must be de-trended

(b) Write stationary planner's problem as **recursive Bellman equation**

(c) Calibrate parameters
   - Find α using production function
   - Find δ by imposing steady state on investment-output ratio
   - Find A using marginal product of capital
   - Find β using steady-state Euler equation (need to find α, δ, A)
   - **Caution:** only allocations *per effective worker* attain steady state

Problem 2 (cont.)

(d) Solve Bellman using **value function iteration**
   - Slides and code on eLC for your reference

(e) Simulate capital, consumption, and output **per effective worker**
   - Assume $k_0 = 0.5 \times k_{ss} = 0.5$ (normalized $k_{ss} = 1$ in (c))
   - Interpolate on $g(k)$ to find $k_1, k_2, \ldots$
   - Use production function, capital to get output
   - Use feasibility, output, and capital to get consumption
   - Check your work
      - Is capital per effective worker converging to steady state?
      - What is capital-output ratio in final period?