TA Session 12

Michael Kotrous
John Munro Godfrey, Sr. Department of Economics
University of Georgia

ECON 8040: Macroeconomics I
November 14, 2025

Revised November 14, 2025

Today's Session

- **Problem Set 6** grades to be posted soon
- **Problem Set 7** due **Friday, November 21** at 11:59 p.m.
- **Computation Exercise 2** due **Tuesday, November 25** at 11:59 p.m.

Problem Set 7

- Due Friday, Nov. 21 at 11:59 p.m.
- Five problems
  - Multi-sector production economy
  - Dynamic model w/ endogenous mortality
  - Asset pricing model
  - Two past exam problems
- This is a long problem set. Don't delay starting it!
- If stuck on one problem, move onto next one (order doesn't matter)

Problem 1

Infinite-horizon production economy w/ business and home sectors

(a) Write down planner's problem
  - Carefully write down all choice variables
  - Output of the business sector can be consumed, or invested in business or home capital
  - Home-produced consumption good cannot be invested

(b) Write the Bellman equation
  - Carefully write down state and control variables
  - Replace constraints in objective where convenient
  - Don't forget to write down all remaining constraints!

Problem 1 (cont.)

(c) Write down FOCs and envelope conditions

- If derivatives are too difficult, adjust Bellman's objective, constraints
- Applying chain rule leads to many cancellations
- To simplify algebra, may be helpful to define

$$Z \equiv \mu c_M^{\rho} + (1-\mu)[k_H^{\alpha}(1-N_M)^{1-\alpha}]^{\rho}$$

(d) Write equations that characterize the steady state

- Feasibility and first-order conditions in steady state
- There should be as many equations as "unknowns." E.g., if you write Bellman w/ four controls, you need to provide four equations in those four unknowns.

Problem 2

Model of endogoneous mortality from Hall and Jones (2007)

(a) Write down planner's problem
   - **Assume discount factor $\beta = 1$**
   - Be careful with choice variables, feasibility and **law of motion**

(b) Write planner's problem recursively. Does it satisfy Blackwell's sufficient conditions?
   - What is the state variable? i.e., what does planner need to know at beginning of period to make optimal allocations?
   - Write down feasibility and law of motion constraints

(c) Show that value function of form vN solves the Bellman. What is v?

Problem 2 (cont.)

(d) Find p

- Replace v from (c) into Bellman
- Write down and combine FOCs

(e) Use given $u(c)$ and $f(h)$ to find health expenditure share of income

- Find eqn. that **implicitly** defines $s \equiv \frac{ph}{y}$ as function of parameters

$$R(s, \alpha, b, \sigma, y) = 0$$

(f) Comparative statics: how does s change in parameters, p?

- **Implicit function theorem**: For given parameter θ,

$$\frac{\partial s}{\partial \theta} = -\frac{\partial R / \partial \theta}{\partial R / \partial s}$$

Problem 3

The Lucas Tree. (Maybe) a helpful analogy: Imagine a shipwrecked crew on an island. The only consumption good available to crew is fruit of a tree on the island. Each period t tree produces fruit d_t. Crew member with s_t shares of tree is entitled to $s_t d_t$ fruit, and shares are traded at price p_t^s. Crew can also trade one-period risk-free bonds b_t at price p_t^b.

The budget constraint is

$$c_t + p_t^b b_{t+1} + p_t^s (s_{t+1} - s_t) = b_t + s_t d_t$$

Problem 3 (cont.)

(a) Define SMCE. State *all* allocations and market clearing conditions. Write utility as $u(c)$.

(b) Write Recursive Competitive Equilibrium.

- Rewrite budget constraint and use hint that $w \equiv b + s(p^s(d) + d)$
- Allocations in (a) are *policy functions*
- Prices in (a) are functions of dividends
- State variables: (w, d)
- Control variables: (c, s', b')
- Refer to example in lecture notes
- **Hint:** You don't need to do part (b) to solve parts (c) and (d)

Problem 3 (cont.)

- Use SMCE definition in (a) to find p_t^s, p_t^b w/ different dividends d_t

(c) Assume $d_t = 1$

- No utility form provided. Just keep utility as $u(c)$

(d) Assume $u(c) = \log c$ and $d_t = \begin{cases} 1 & t = 0, 2, 4, \dots \\ 2 & t = 1, 3, 5, \dots \end{cases}$

- Stock, bond prices each take two values for even/odd periods

Problem 4

Growth model on Fall 2017 Final Exam

- **Advice:** Treat this problem like practice final exam. Set aside your notes and try to do this problem in ≤ 3 hours

(a) Write down aggregate feasibility (in terms of aggregate variables (e.g., K_t) or *per person* variables (e.g., $k_t \equiv \frac{K_t}{(1+\eta)^t}$))

(b) Write down firm's profit maximization problem. Derive formulas for rental rate of capital r_t and wage w_t

(c) Write down expression for national income

(d) Define sequential markets competitive equilibrium

Problem 4 (cont.)

(e) Write down three equations using *per person* variables:

- Euler equation
- Aggregate feasibility
- Intratemporal optimality for consumption−leisure

(f) Write equilibrium interest rate i_t in terms of capital-labor ratio, tax and parameters

- FOC with respect to a_{t+1}

(g) What is the long-run growth rate of *per person* variables? What is the long-run growth rate of hours per person?

- Don't overthink this one

(g) Detrend all **per person variables** appropriately. Denote de-trended variables with $\widehat{\ }$ (e.g., $\widehat{k}$). Rewrite aggregate feasibility with hats.

Problem 4 (cont.)

(h) Write down three equations in steady state:

1. Euler equation
2. Aggregate feasibility
3. Marginal rate of substitution between consumption and leisure

   – Why are $\widehat{k}$ and other variables constant in the long-run?

- Desribe the long-run effect of τ on

  (i) interest rate i_t

  (j) capital per worker $\widehat{k}/h$

(k) Calibrate the parameters $(\delta, \alpha, \phi, \beta, \tau)$ model using provided table

   – Recommended order: τ, α, ϕ, δ, β

Problem 5

Macro Preliminary Exam, Summer 2018

(a) Define Arrow-Debreu competitive equilibrium.
   - Write down all prices and allocations
   - Be careful writing budget constraints for workers and capitalists
   - Don't forget policy variables and government budget equation

(b) Solve for n_t^1
   - Take first order conditions for workers
   - Then use budget constraint

(b) Write c_t^1, c_t^2 in terms of post-tax labor income and post-tax dividend

Problem 5 (cont.)

(c) Solve for q_t in terms of q_{t+1}, d_{t+1}, and consumption growth
  - FOC with respect to s_{t+1}

(d) Firm's first-order conditions with respect to choice variables

(e,f,g) Find steady state capital, wage, and stock price q_t

(h,i) Comment on effects of tax reform

Computation Exercise 2

- Due Tuesday, Nov. 25 at 11:59 p.m.
- Solve finite and infinite horizon models numerically
- Submit Matlab code that **runs**
  - Functions defined at bottom of script or in separate file
  - You should be able to open your m file and click “Run” w/out error
  - Put all plots and tables asked for in your PDF

Problem 1

Two-period planning problem

(a) Use **Newton's Method** to find (c_1, c_2, k_2) when $\bar{k}_1 = k_{ss}$

- Calculate k_{ss} using using provided parameter values
- Rewrite three equations s.t. all equal 0
- Write 3×3 Jacobian matrix
- Refer to Newton's Method code you wrote for CE1

(b) Find allocations, welfare for other initial capital stocks

- Do it for all $\mathcal{K} = \{0.5k_{ss}, 0.75k_{ss}, k_{ss}, 1.5k_{ss}, 2k_{ss}\}$
- Plug c_1, c_2 in utility function to find $w(\bar{k}_1)$

Problem 2

Infinite-horizon planning problem w/ pop. and productivity growth

(a) Rewrite planner's problem so feasibility constraint is stationary
  - Divide aggregates by growth rates to get **per effective worker allocations**
  - **Caution:** Per person consumption in utility must be de-trended

(b) Write stationary planner's problem as **recursive Bellman equation**

(c) Calibrate parameters
  - Find α using production function
  - Find δ by imposing steady state on investment-output ratio
  - Find A using marginal product of capital
  - Find β using steady-state Euler equation (need to find α, δ, A)
  - **Caution:** only allocations *per effective worker* attain steady state

Problem 2 (cont.)

(d) Solve Bellman using **value function iteration**

- Slides and code on eLC for your reference

(e) Simulate capital, consumption, and output **per effective worker**

- Assume $k_0 = 0.5 \times k_{ss} = 0.5$ (normalized $k_{ss} = 1$ in (c))
- Interpolate on $g(k)$ to find $k_1, k_2, \ldots$
- Use production function, capital to get output
- Use feasibility, output, and capital to get consumption
- Check your work
  - Is capital per effective worker converging to steady state?
  - What is capital-output ratio in final period?