

ECON 8040 – TA9

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Today's Session

- ★ PS5 due today (Friday, October 20)
- ★ PS6 due Thursday, October 26
- ★ PS3 and PS4 grades posted to eLC

Problem 1

Ag, manufacturing, services share model

- a) Set up similar to PS4, Problem 2
- b) Pull data for ag, manufacturing, and services consumption in U.S.
- c) “Calibrate” the parameters of the model
- d) Interpret \bar{c}_a and \bar{c}_s parameters
- e) Compare expenditure shares path predicted by model against actual data

Problem 2

Define competitive equilibrium for static model of continuous labor supply

- ★ State all equilibrium objects
- ★ Define household problem
 - Labor and leisure must add up to time endowment
- ★ Define firm problem
- ★ Three markets clear

Problem 3

Solve static model of continuous labor supply, assuming

$$u(c, \ell) = \frac{1}{1 - \sigma} \left[\left(c^\phi \ell^{1-\phi} \right)^{1-\sigma} - 1 \right] \quad F(K, N) = AK^\alpha N^{1-\alpha}$$

- ★ Assume $\phi \in (0, 1)$, $\sigma \geq 1$
- ★ You may assume $\bar{k} = 1$ (but you don't have to)
- ★ What is k^* ?
- ★ Rewrite HH problem in terms of two choices: c, ℓ
- ★ Write MRS of c, ℓ
- ★ Use budget constraint, firm FOCs to write equation with one unknown: ℓ^*
- ★ Use ℓ^* to solve other allocations, prices

Problem 4

Solve static model of continuous labor supply, assuming

$$u(c, \ell) = c - \frac{1}{1 + \frac{1}{\varepsilon}} (1 - \ell)^{1 + \frac{1}{\varepsilon}} \qquad F(K, N) = AK^\alpha N^{1-\alpha}$$

- ★ Assume $\varepsilon > 0$
- ★ Write HH problem with two choices: c, h (or c, ℓ)
- ★ Use FOCs to write w in terms of h
- ★ Use firm's FOC to write equation with one unknown: h^*
- ★ Use h^* to find other allocations, prices

Problem 1

- a) Write the Bellman equation
- b) Solve the Bellman using the provided guess and following the lecture notes
 - Solve policy function k' and value function $v(k)$
- c) Use the policy function for k' to find $\frac{k_{t+1}}{k_t}$ and $\frac{c_{t+1}}{c_t}$

Problem 2

- a) Write down FOC for n using $F(k, k')$
Write Bellman equation using $F(k, k')$
- b) Assume full depreciation ($\delta = 1$)
Find FOC for k' using provided guess: $V(k) = A + B \log(k)$
- c) Write n in terms of parameters and B
- d) Replace k' and n in guess of $V(k)$ to solve B
- e) Solve for policy functions n , k' , and c as function of state k

Problem 3

- a) Write planning problem $w(k_0, h_0)$ (k_0, h_0 is given initial capital stock)
- b) Write the planning problem recursively
- c) Assume full depreciation ($\delta = 1$) and use guess-and-verify to solve:
 - $V(k, h)$
 - $k'(k, h)$
 - $h'(k, h)$

Problem 4

- a) Rewrite the problem so $\{k_{t+1}\}_{t=0}^{\infty}$ is only choice variable
- b) Write the problem recursively using two equations
 - $v(k, \theta_L)$
 - $v(k, \theta_H)$
 - You know how state θ_t evolves
- c) Solve the Bellman equations using guess-and-verify
- d) Find policy functions $g(k, \theta_L)$ and $g(k, \theta_H)$

Problem 1

- a) Want to show result for *any* size distribution
- c) With CRS technology, firms earn **zero** profit in equilibrium

Problems 2,4,5

Penn World Table data exercises

- ★ Cross-country differences in income, capital, productivity
- ★ Cross-country differences in income, savings rates, productivity
- ★ Solow model and “catch-up” growth
 - Assume U.S. is in steady state
 - Assume “your country” has same α , A , δ , s
 - \Rightarrow “your country” converges to k_{US}

Problem 1

Static model of indivisible labor supply (i.e., $h = 0$ or $h = 1$)

a) Define competitive equilibrium

- Use i superscript for HH allocations (no representative HH)
- HH makes allocation k^{i*} . Optimal choice is $k^{i*} = \bar{k}^i$
- HH chooses h^{i*} , not ℓ^{i*}
- WLOG normalize $p^* = 1$
- Firm has CRS technology
 - Representative firm
 - Zero profit in equilibrium

Problem 2

Static model of expenditure shares agriculture, manufacturing, goods

- a) Find expenditure shares for each good, i.e find $\frac{p_i c_i}{y}$

$$\max_{\{c_a, c_m, c_s\}} \phi_a \log(c_a - \bar{c}_a) + \phi_m \log(c_m) + \phi_s \log(c_s + \bar{c}_s)$$

subject to $p_a c_a + p_m c_m + p_s c_s = y; \lambda$

Sketch:

- Divide FOC wrt c_a by FOC wrt c_m
- Divide FOC wrt c_s by FOC wrt c_s
- Write $\frac{p_a c_a}{y}, \frac{p_s c_s}{y}$ in terms of parameters, $\frac{p_m c_m}{y}$
- Divide budget constraint by y ; replace $\frac{p_a c_a}{y}, \frac{p_s c_s}{y}$
- Solve for $\frac{p_m c_m}{y}$ in terms of parameters
- Replace $\frac{p_m c_m}{y}$ in equations in step 3 to finish

Problem 4

Two-period sequential market economy

The correct endowments are:

$$(e_0^1, e_1^1) = (1, 0)$$

$$(e_0^2, e_1^2) = (0, 1)$$

See Fall 2021 Midterm Solutions document for answer