ECON 8040 – TA8

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Today's Session

- ⋆ Midterm bonus points assigned
- ⋆ PS4 due today (Friday, October 13) at 11:59pm
  - ⟶ PS4, Problem 3 is optional
- ⋆ **PS5 extended to Friday, October 20**
- ⋆ PS6 due Friday, October 27

Problem 1

Static model of indivisible labor supply (i.e., $h = 0$ or $h = 1$)

a) Define competitive equilibrium
   - $\rightarrowtail$ Write household problem for all $i \in [0, 1]$
   - $\rightarrowtail$ Write firm problem
   - $\rightarrowtail$ 3 market-clearing conditions
     - integrate over allocations by households on $[0, 1]$ to get aggregates

b) In equilibrium, households are *indifferent* between working full-time and not working at all
   1. Use this condition to write down an equation. (Think carefully about how much workers/non-workers consume.)
   2. Solve for $\frac{r^*}{w^*}$ (it equals a constant)
   3. Write down firm's FOCs
   4. Combine expression from steps 2 and 3 to write an equation that has equilibrium labor supply n^* as its only variable and solve.

Problem 2

Static model of expenditure shares agriculture, manufacturing, goods

a) Find expenditure shares for each good, i.e find $\frac{p_i c_i}{y}$

   1) Write down utility maximization subject to budget constraint.
   2) FOCs with respect to decision variables. This gives you three equations with three unknowns.
   3) Solve for $\frac{p_i c_i}{y}$ for $i \in \{a, m, s\}$.

b) How do expenditures shares change as you increase y?

   1) Check sign of $\frac{\partial\left(\frac{c_i p_i}{y}\right)}{\partial y}$ for $i \in \{a, m, s\}$
   2) Sanity check: Do your results match your intuition about economic development?

Problem 3

a) **(Optional)** Finite horizon planning problem
   - ⟶ Write Euler equation
   - ⟶ Rearrange so that left-hand side of equation is

$$z_{t+1} \equiv \frac{k_{t+2}}{Ak_{t+1}^{\alpha}}$$

     and $z_t \equiv \frac{k_{t+1}}{Ak_t^{\alpha}}$ is on the right-hand side
   - ⟶ Solve for z_t in terms of parameters and z_{t+1}.
   - ⟶ Start in final period and work backward (i.e., $k_{T+1} = 0 \Rightarrow z_T = 0$)
     - Why is this the case?
   - ⟶ Notice pattern and write equation for z_t

b) Evaluate limits

Problem 4

Two-period sequential market economy

The correct endowments are:

$$(e_0^1, e_1^1) = (1, 0)$$
$$(e_0^2, e_1^2) = (0, 1)$$

Problem 4

Two-period sequential market economy

a) Define SMCE
   - $\rightarrowtail$ Household has separate budget constraints for two periods
   - $\rightarrowtail$ Be careful with η when defining market clearing conditions

b) Find equilibrium interest rate i^* as function of η
   - $\rightarrowtail$ FOCs wrt c_0^k, c_1^k, a^k

c) Discuss why interest rate changes as it does when η increases. Evaluate

$$\frac{\partial i^*}{\partial \eta}$$

Problem 1

Ag, manufacturing, services share model

a) Set up similar to PS4, Problem 2

b) Pull data for ag, manufacturing, and services consumption in U.S.

c) "Calibrate" the parameters of the model

d) Interpret $\bar{c}_a$ and $\bar{c}_s$ parameters

e) Compare expenditure shares path predicted by model against actual data

Problem 2

Define competitive equilibrium for static model of continuous labor supply

- ⋆ State all equilibrium objects
- ⋆ Define household problem
  - ⟶ Labor and leisure must add up to time endowment
- ⋆ Define firm problem
- ⋆ Three markets clear

Problem 3

Solve static model of continuous labor supply, assuming

$$u(c,\ell) = \frac{1}{1-\sigma}\left[\left(c^{\phi}\ell^{1-\phi}\right)^{1-\sigma} - 1\right] \qquad F(K,N) = AK^{\alpha}N^{1-\alpha}$$

- ⋆ Assume $\phi \in (0,1)$, $\sigma \geq 1$
- ⋆ You may assume $\bar{k} = 1$ (but you don't have to)
- ⋆ What is k^*?
- ⋆ Rewrite HH problem in terms of two choices: c, ℓ
- ⋆ Write MRS of c, ℓ
- ⋆ Use budget constraint, firm FOCs to write equation with one unknown: ℓ^*
- ⋆ Use ℓ^* to solve other allocations, prices

Problem 4

Solve static model of continuous labor supply, assuming

$$u(c, \ell) = c - \frac{1}{1 + \frac{1}{\varepsilon}}(1 - \ell)^{1+\frac{1}{\varepsilon}} \qquad F(K, N) = AK^{\alpha}N^{1-\alpha}$$

- ⋆ Assume $\varepsilon > 0$
- ⋆ Write HH problem with two choices: c, h (or c, ℓ)
- ⋆ Use FOCs to write w in terms of h
- ⋆ Use firm's FOC to write equation with one unknown: h^*
- ⋆ Use h^* to find other allocations, prices

Problem 1

(a) Write the Bellman equation

(b) Solve the Bellman using the provided guess and following the lecture notes

$\rightarrowtail$ Solve policy function k' and value function $v(k)$

(c) Use the policy function for k' to find $\frac{k_{t+1}}{k_t}$ and $\frac{c_{t+1}}{c_t}$

Problem 2

(a) Write down FOC for n using $F(k, k')$
Write Bellman equation using $F(k, k')$

(b) Assume full depreciation ($\delta = 1$)
Find FOC for k' using provided guess: $V(k) = A + B\log(k)$

(c) Write n in terms of parameters and B

(d) Replace k' and n in guess of $V(k)$ to solve B

(e) Solve for policy functions n, k', and c as function of state k

Problem 3

(a) Write planning problem $w(k_0, h_0)$ (k_0, h_0 is given initial capital stock)

(b) Write the planning problem recursively

(c) Assume full depreciation ($\delta = 1$) and use guess-and-verify to solve:

- $\rightarrowtail V(k, h)$
- $\rightarrowtail k'(k, h)$
- $\rightarrowtail h'(k, h)$

Problem 4

(a) Rewrite the problem so $\{k_{t+1}\}_{t=0}^{\infty}$ is only choice variable

(b) Write the problem recursively using two equations
   - $\rightarrowtail$ $v(k, \theta_L)$
   - $\rightarrowtail$ $v(k, \theta_H)$
   - $\rightarrowtail$ You know how state θ_t evolves

(c) Solve the Bellman equations using guess-and-verify

(d) Find policy functions $g(k, \theta_L)$ and $g(k, \theta_H)$

Matlab Installation

- ⋆ Install Matlab
  - ⟼ Free through UGA
  - ⟼ UGA IT installation guide
  - ⟼ Value function iteration, computational exercises require Matlab
  - ⟼ ECON 8050 also requires Matlab
  - ⟼ Matlab coding usually tested on macro preliminary exam