ECON 8040 – TA7

Michael Kotrous

University of Georgia

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Today's Session

- ⋆ Midterm Retake (bonus credit) due October 6 (today) at 11:59pm
  - ⟼ Those who redo midterm problems satisfactorily will receive **15 points** added to their midterm score
- ⋆ PS4 extended to Friday, October 13 at 11:59pm
  - ⟼ Problem 3 is optional
- ⋆ PS5 due Friday, October 13 at 11:59pm

General Thoughts

- ⋆ Read description of model carefully!
- ⋆ Models differed from HW and lecture notes
  - ⟶ Thus, correct answers on exam don't match HW problems
  - ⟶ When studying lecture notes / HW, learn solution methods, not just results

Problem 1

a) Define ADCE

$\rightarrowtail$ State all equilibrium objects *first*

$$\{c_t^1, c_t^2, p_t\}_{t=0}^{\infty}$$

$\rightarrowtail$ Given prices, household i chooses only *own* consumption

$$\max_{\{c_t^1, c_t^2\}_{t=0}^{\infty}} \cancel{U(c_t)} \qquad \max_{\{c_t^i\}_{t=0}^{\infty}} U(c_t)$$

$\rightarrowtail$ Household has only one budget constraint

$\rightarrowtail$ Market clears every period

Problem 1

b) Define Pareto efficient allocation
   - ⟶ Define "feasible"
   - ⟶ *Do not define a Planner's Problem*

c) Prove first welfare theorem
   - ⟶ Proof by contradiction (i.e., show $CE \wedge \neg PE$ is wrong)
   - ⟶ Follow sketch in lecture notes, Proposition 2 on page 7 of "Introduction to Competitive Equilibria and Welfare Theorems"

Problem 1

d) Define Planner Problem

$\rightarrowtail$ Planner does not face budget constraint

e,f,g) Plug in the correct endowments!

$$e_t^1 = \begin{cases} 2 & \text{if } t = 0, 2, 4, \ldots \\ 0 & \text{if } t = 1, 3, 5, \ldots \end{cases}$$

$$e_t^2 = \begin{cases} 0 & \text{if } t = 0, 2, 4, \ldots \\ 1 & \text{if } t = 1, 3, 5, \ldots \end{cases}$$

g) Find equilibrium prices

$\rightarrowtail$ Don't write down ADCE, use Negishi Method

Problem 2

a) Detrend the aggregate feasibility constraint
   - $\rightarrowtail$ Replace values in aggregate feasibility equation
   - $\rightarrowtail$ Divide both side by $N' = (1 + n)N$

b) Use the equation from a)
   - $\rightarrowtail$ Impose $k^* \equiv k' = k$ and do algebra
   - $\rightarrowtail$ Hint: $sy \neq \delta k$ in this model

c) Plot evolution of aggregate output Y_t over time
   - $\rightarrowtail$ What goes on y-axis?
   - $\rightarrowtail$ What goes on x-axis?
   - $\rightarrowtail$ Use Excel if you must

Problem 1

Static model of indivisible labor supply (i.e., $h = 0$ or $h = 1$)

a) Define competitive equilibrium
   - $\rightarrowtail$ Write household problem for all $i \in [0, 1]$
   - $\rightarrowtail$ Write firm problem
   - $\rightarrowtail$ 3 market-clearing conditions
     - integrate over allocations by households on $[0, 1]$ to get aggregates

b) In equilibrium, households are *indifferent* between working full-time and not working at all
   1. Use this condition to write down an equation. (Think carefully about how much workers/non-workers consume.)
   2. Solve for $\frac{r^*}{w^*}$ (it equals a constant)
   3. Write down firm's FOCs
   4. Combine expression from steps 2 and 3 to write an equation that has equilibrium labor supply n^* as its only variable and solve.

Problem 2

Static model of expenditure shares agriculture, manufacturing, goods

a) Find expenditure shares for each good, i.e find $\frac{p_i c_i}{y}$

   1) Write down utility maximization subject to budget constraint.
   2) FOCs with respect to decision variables. This gives you three equations with three unknowns.
   3) Solve for $\frac{p_i c_i}{y}$ for $i \in \{a, m, s\}$.

b) How do expenditures shares change as you increase y?

   1) Check sign of $\frac{\partial\left(\frac{c_i p_i}{y}\right)}{\partial y}$ for $i \in \{a, m, s\}$
   2) Sanity check: Do your results match your intuition about economic development?

Problem 3

a) **(Optional)** Finite horizon planning problem
   - ⟶ Write Euler equation
   - ⟶ Rearrange so that left-hand side of equation is

$$z_{t+1} \equiv \frac{k_{t+2}}{Ak_{t+1}^{\alpha}}$$

     and $z_t \equiv \frac{k_{t+1}}{Ak_t^{\alpha}}$ is on the right-hand side
   - ⟶ Solve for z_t in terms of parameters and z_{t+1}.
   - ⟶ Start in final period and work backward (i.e., $k_{T+1} = 0 \Rightarrow z_T = 0$)
     - Why is this the case?
   - ⟶ Notice pattern and write equation for z_t

b) Evaluate limits

Problem 4

Two-period sequential market economy

The correct endowments are:

$$(e_0^1, e_1^1) = (1, 0)$$
$$(e_0^2, e_1^2) = (0, 1)$$

Problem 4

Two-period sequential market economy

a) Define SMCE
   - $\rightarrowtail$ Household has separate budget constraints for two periods
   - $\rightarrowtail$ Be careful with η when defining market clearing conditions

b) Find equilibrium interest rate i^* as function of η
   - $\rightarrowtail$ FOCs wrt c_0^k, c_1^k, a^k

c) Discuss why interest rate changes as it does when η increases. Evaluate

$$\frac{\partial i^*}{\partial \eta}$$

Problem 1

Ag, manufacturing, services share model

a) Set up to PS4, Problem 2
b) Pull data for ag, manufacturing, and services consumption in U.S.
c) "Calibrate" the parameters of the model
d) Interpret $\bar{c}_a$ and $\bar{c}_s$ parameters
e) Compare expenditure shares path predicted by model against actual data

Problem 2

Static model of continuous labor supply

2. Define competitive equilibrium
   - ⟶ State all equilibrium objects
   - ⟶ Define household problem
     - Labor and leisure must add up to time endowment
   - ⟶ Define firm problem
   - ⟶ Three markets clear

Problems 3, 4

Solve static model of continuous labor supply

3. Assume

$$u(c,\ell) = \frac{1}{1-\sigma}\left[\left(c^{\phi}\ell^{1-\phi}\right)^{1-\sigma} - 1\right] \qquad F(K,N) = AK^{\alpha}N^{1-\alpha}$$

   - $\rightarrowtail$ Assume $\phi \in (0,1)$
   - $\rightarrowtail$ You may assume $\bar{k} = 1$ (but you don't have to)

4. Assume

$$u(c,\ell) = c - \frac{1}{1+\frac{1}{\varepsilon}}(1-\ell)^{1+\frac{1}{\varepsilon}} \qquad F(K,N) = AK^{\alpha}N^{1-\alpha}$$

   - $\rightarrowtail$ Assume $\varepsilon > 0$
   - $\rightarrowtail$ Easier to solve $h^* = 1 - \ell^*$