ECON 8040 – TA Session 3

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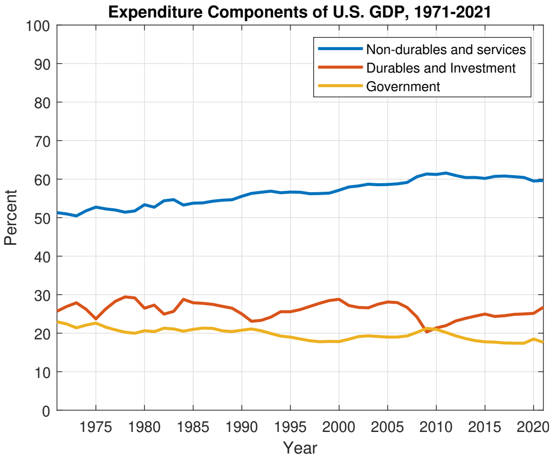
Today's Session

- ⋆ Homework 2 submitted
- ⋆ Homework 1 grades

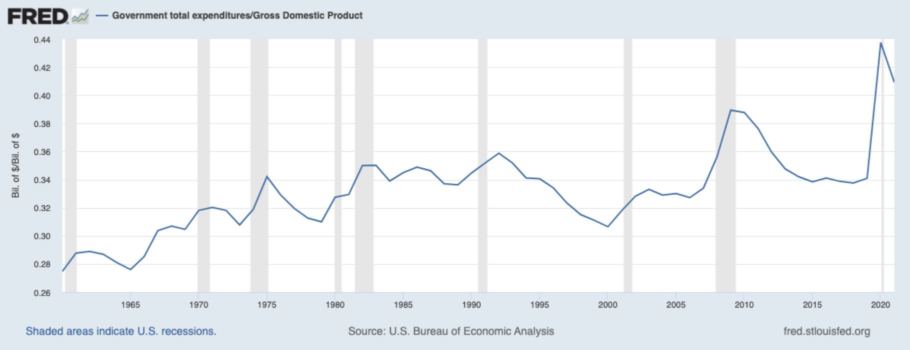
Problem 1, parts a,b,c



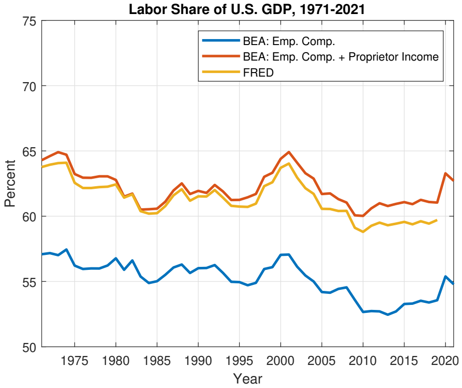
⋆ Sanity check: Should sum close to 1 (GDP $\approx C + I + G$ for NX small)

Problem 1, part c

- Alternative calculation: Gov't Share $\equiv \frac{\text{Gov't Total Exp.}}{\text{GDP}}$

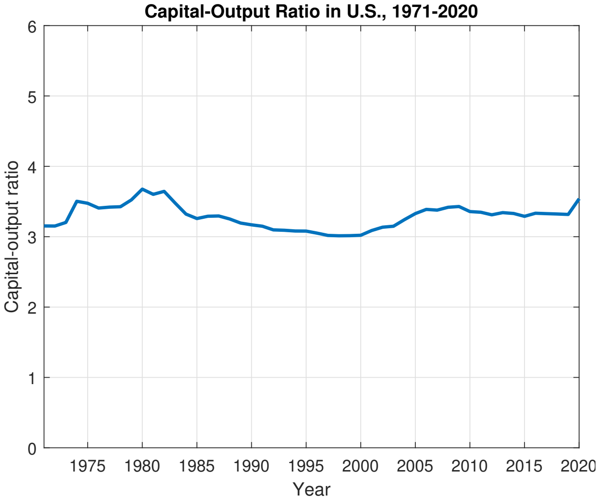


Problem 1, part d

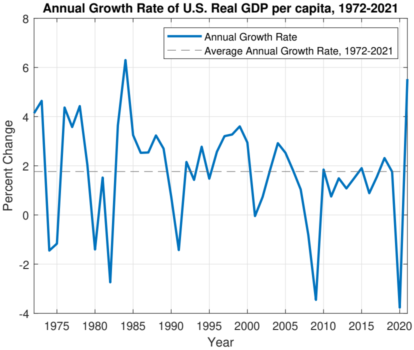


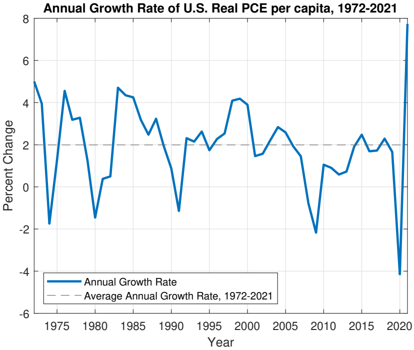
- ⋆ Recent papers on declining labor share: "Superstar Firms" (Autor et. al, 2020); "Rise of Market Power" (De Loecker, Eeckhout, and Unger, 2020)

Problem 1, part e



Problem 1, parts f,g





Problem 2

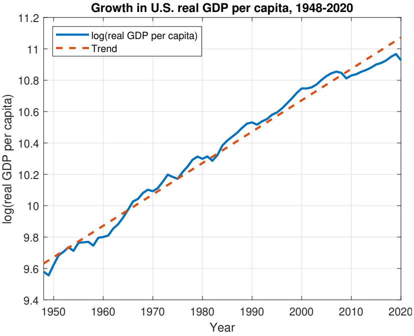
Report trends and residuals for data series:

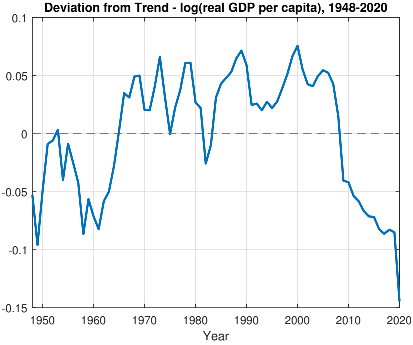
1. Generate $\log(x)$
2. Fit linear trend $\widehat{\log(x)}$ that minimizes sum of squared errors
3. Compute "Deviations" / "residuals": $\hat{e} = \log(x) - \widehat{\log(x)}$

How I did it in Matlab:

1. Compute $\log(x)$
2. Matlab's `detrend` function fits line and returns residuals $\hat{e}$
3. Trend $\widehat{\log(x)} = \log(x) - \hat{e}$

Problem 2, part a





Problem 3

Prove properties of CRRA utility

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

a) Let $f(\sigma) = c^{1-\sigma} - 1$, $g(\sigma) = 1 - \sigma$. Both are differentiable and $\lim_{\sigma \to 1} \frac{f(\sigma)}{g(\sigma)} = \frac{0}{0}$. By l'Hôspital's rule,

$$\lim_{\sigma \to 1} U(c) = \lim_{\sigma \to 1} \frac{f'(\sigma)}{g'(\sigma)} = \log c$$

b) Plug derivatives into definition to show

$$-\frac{U''(c)}{U'(c)} c = \sigma$$

Problem 3

c) Plug into definition

$$\text{IES} \equiv -\frac{\%\Delta c}{\%\Delta U'(c)} = -\frac{dc}{c} \Bigg/ \frac{dU'(c)}{U'(c)} = -\frac{U'(c)}{c\left(\frac{dU'(c)}{dc}\right)} = -\frac{U'(c)}{c \times U''(c)}$$

$\rightarrowtail$ Sensitivity of intertemporal substitution wrt 1% change in MRS

d) Use $U'(c), U''(c)$ to show Inada conditions

i. strictly increasing
ii. strictly concave
iii. $\lim_{c \to 0} U'(c) = +\infty$
iv. $\lim_{c \to +\infty} U'(c) = 0$

Problem 3

e) Show MRS is invariant to scaling of consumption.

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

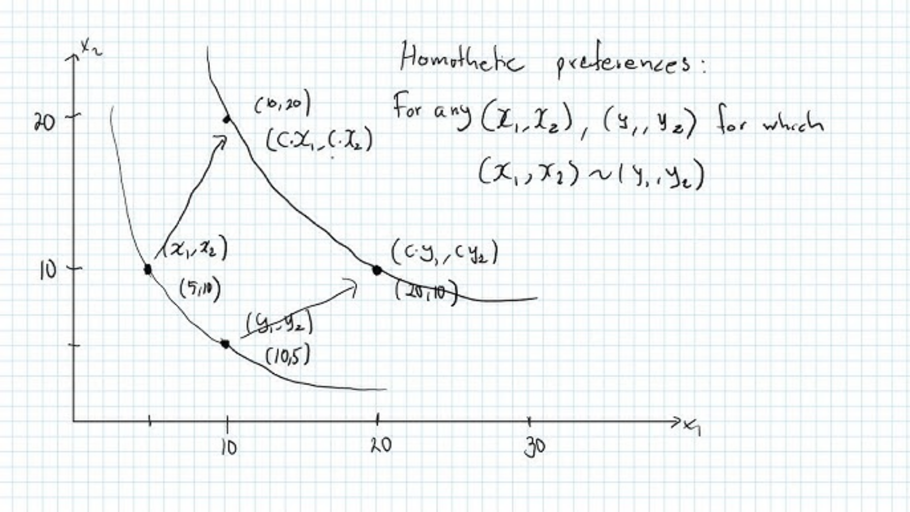
subject to $\sum_{t=0}^{\infty} p_t c_t \leq \sum_{t=0}^{\infty} p_t e_t; \gamma$
FOCs wrt c_{t+s}, c_t

- $\beta^{t+s} c_{t+s}^{-\sigma} = \gamma p_{t+s}$
- $\beta^t c_t^{-\sigma} = \gamma p_t$

For $\lambda > 0$,

$$MRS(c_{t+s}, c_t) = \beta^s \left(\frac{c_{t+s}}{c_t}\right)^{-\sigma} = MRS(\lambda c_{t+s}, \lambda c_t)$$

Problem 3, part e



Problem 3

f) Given $\{\bar{c}_t\}_{t=0}^{\infty}$ is CE allocation when income equals y. Need to show allocation $\{\widetilde{c}_t\}_{t=0}^{\infty} = \{\lambda \bar{c}_t\}_{t=0}^{\infty}$ when income $\widetilde{y} = \lambda \bar{y}$

- Affordable using the budget constraint
- Optimal using homotheticity of CRRA utility (i.e., MRS invariant to scaling of consumption)

Problem 4

First *three* questions from the lecture notes

1. Arrow-Debreu CE is allocation $\{c_t^1, c_t^2\}_{t=0}^{\infty}$ and prices $\{p_t\}_{t=0}^{\infty}$ such that
   A. Given prices, household $i = 1, 2$ makes allocation that solves

$$\max_{\{c_t^i\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_t^i$$

   s.t. $\sum_{t=0}^{\infty} p_t c_t^i = \sum_{t=0}^{\infty} p_t e_t^i$

   B. For $t = 0, 1, 2, \ldots,$ market clears

$$c_t^1 + c_t^2 = e_t^1 + e_t^2$$

   where $e_t^1 = (2, 0, 2, 0, \ldots)$ and $e_t^2 = (0, 2, 0, 2, \ldots)$

Problem 4

First *three* questions from the lecture notes

a) Show prices are *not* constant. E.g., prove by contradiction

- Suppose $p_t = p$ for $t = 0, 1, 2, \ldots$

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log(c_t^i)$$

  s.t. $\sum_{t=0}^{\infty} pc_t^i = \sum_{t=0}^{\infty} pe_t^i; \lambda$

- By FOCs, $\lambda = \frac{\beta^t}{c_t^i} = \frac{\beta^{t+1}}{c_{t+1}^i} \Leftrightarrow c_t^i = \beta c_{t-1}^i = \beta^t c_0^i$
- By budget constraint, $\sum_{t=0}^{\infty} \beta^t pc_0^i = \frac{pc_0^i}{(1-\beta)} = \sum_{t=0}^{\infty} pe_t^i \Leftrightarrow c_0^i = (1-\beta) \sum_{t=0}^{\infty} e_t^i$
- Contradiction: HH cannot consume infinite resources in first period

b) Solve ADCE

- Derive Euler equation using household i's first-order conditions

$$c_{t+1}^i = \beta \frac{p_t}{p_{t+1}} c_t^i$$

- Write c_t^i in terms of c_0^i using Euler

$$c_t^i = \beta \frac{p_{t-1}}{p_t} c_{t-1}^i = \beta \frac{p_{t-1}}{p_t} \times \beta \frac{p_{t-2}}{p_{t-1}} c_{t-2}^i = \cdots = \beta^t \frac{p_0}{p_t} c_0^i$$

- Normalize $p_0 = 1$. Write c_0^i in terms of discount factor, prices, and endowments using budget constraint

$$\sum_{t=0}^{\infty} p_t c_t^i = \sum_{t=0}^{\infty} p_t \beta^t \frac{p_0}{p_t} c_0^i = c_0^i \sum_{t=0}^{\infty} \beta^t = \sum_{t=0}^{\infty} p_t e_t^i$$

- Replace c_0^i in equation you wrote earlier for c_t^i

$$c_t^i = (1-\beta) \frac{\beta^t}{p_t} \sum_{t=0}^{\infty} p_t e_t^i$$

b) Solve ADCE

- Solve p_t using the market-clearing condition

$$c_t^1 + c_t^2 = (1-\beta)\frac{\beta^t}{p_t}\sum_{t=0}^{\infty} p_t(e_t^1 + e_t^2)$$

$$2 = 2(1-\beta)\frac{\beta^t}{p_t}\sum_{t=0}^{\infty} p_t$$

$$p_t = \beta^t$$

- Solve allocations $\hat{c}_t^1$, $\hat{c}_t^2$ by plugging p_t, endowments in expression for c_t^i (be careful with sums!)

$$\hat{c}_t^i = (1-\beta)\sum_{t=0}^{\infty} \beta^t e_t^i$$

Problem 4

2. Prove equivalence of ADCE and SMCE
   Exercise (optional): Show $1 + r_{t+1} = \frac{1}{\beta}$, $c_t^1 = \frac{2}{1-\beta}$, $c_t^2 = \frac{2\beta}{1-\beta}$ in SMCE with same preferences, endowments as ADCE in Question 1.
   SMCE is allocation $\{c_t^1, a_{t+1}^1, c_t^2, a_{t+1}^2\}_{t=0}^{\infty}$ and interest rate $\{r_{t+1}\}_{t=0}^{\infty}$ such that
   A. Given prices, household $i = 1, 2$ makes allocation that solves

$$\max_{\{c_t^i\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_t^i$$

   s.t. $c_t^i + a_{t+1}^i = e_t^i + (1 + r_t)a_t^i$
   B. For $t = 0, 1, 2, \ldots,$ markets clear
      i. Consumption good: $c_t^1 + c_t^2 = e_t^1 + e_t^2$
      ii. Securities: $a_{t+1}^1 + a_{t+1}^2 = 0$

Problem 4

3. Prove First Welfare Theorem: Let $\{c_t^1, c_t^2\}_{t=0}^{\infty}$ be CE. Then it is Pareto efficient.
   Suppose not. If proposition is incorrect, $\exists$ allocation $\{\hat{c}_t^1, \hat{c}_t^2\}_{t=0}^{\infty}$ such that
   i. $\sum_{t=0}^{\infty} \beta^t \log(\hat{c}_t^1) > \sum_{t=0}^{\infty} \beta^t \log(c_t^1)$
   ii. $\hat{c}_t^1 + \hat{c}_t^2 = 2$ for $t = 0, 1, 2, \ldots$
   iii. $\sum_{t=0}^{\infty} \beta^t \log(\hat{c}_t^2) = \sum_{t=0}^{\infty} \beta^t \log(c_t^2)$

   Sketch: Find $\{\hat{c}_t^1, \hat{c}_t^2\}_{t=0}^{\infty}$ s.t i. and ii. hold. Use fact that log preferences are **locally non-satiated** and to conclude $\sum_{t=0}^{\infty} \beta^t \log(\hat{c}_t^2) < \sum_{t=0}^{\infty} \beta^t \log(c_t^2)$, which finishes the proof.