ECON 8040 – TA14

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Today's Session

- ⋆ Final exam **Thursday, December 7, 3:30–6:30 p.m.**
- ⋆ PS8 and PS9 Grades

Problem 1

(a) Define SMCE in "Lucas Tree" economy

A. HH maximizes utility subject to
   - budget constraint
   - no Ponzi schemes

B. Markets clear
   i. Consumption good: $c_t = d_t$ for $t = 0, 1, \dots$
   ii. Shares: $s_t = 1$ for $t = 0, 1, \dots$
   iii. Assets: $b_t = 0$ for $t = 0, 1, \dots$

Problem 1

(b) Recursive Competitive Equilibrium is collection of price functions $\{p^s(d), p^b(d)\}$, value function $\{V^*(w,d)\}$, and household policy functions $\{c^*(w,d), b^*(w,d), s^*(w,d)\}$ such that

A. Given price functions and dividends d and d', household policy functions solve its Bellman equation

$$V(w,d) = \max_{c,b',s'} \{u(c) + \beta V(w', d')\}$$

subject to

$$\begin{aligned} c + p^b(d)b' + p^s(d)s' &\leq w, \\ c, s' &\geq 0 \\ b' &\geq -\bar{A} \text{ for some } \bar{A} > 0 \end{aligned}$$

The solution to the Bellman equation is value function $V^*(w,d)$.

Problem 1

B. Price functions are functions of dividends d.

$$p^s = p^s(d)$$
$$p^b = p^b(d)$$

C. Allocation is feasible. That is, household policies satisfy market clearing conditions in aggregate. For all dividends d,

$$c(d, d) = d$$
$$b(d, d) = 0$$
$$s(d, d) = 1$$

Problem 1

(c) Assume $d_t = 1$ and find p_t^b

- HH FOCs wrt b_{t+1}, c_t, and c_{t+1}

$$p_t^b = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

- By MCC, $c_t = c_{t+1} = 1$, so

$$p_t^b = \beta$$

Problem 1

(c) Assume $d_t = 1$ and find p_t^s

- HH FOCs wrt k_{t+1}, c_t, and c_{t+1}

$$p_t^s = \beta \frac{u'(c_{t+1})}{u'(c_t)} \left(p_{t+1}^s + d_{t+1}\right)$$

- Iterate forward r periods

$$p_t^s = \beta^r p_{t+r}^s + \sum_{k=1}^{r} \beta^k$$

- Assume no speculative bubble: $\lim_{r\to\infty} \beta^r p_{t+r}^s = 0$

$$p_t^s = \sum_{k=1}^{\infty} \beta^k$$

Problem 1

(d) Assume $u(c_t) = \log(c_t)$ and $d_t = \begin{cases} 1 & \text{if } t = 0, 2, 4, \dots \\ 2 & \text{if } t = 1, 3, 5, \dots \end{cases}$

- Solve for p_t^b, p_t^s same as (c)
- Two prices for bonds, stocks (odd/even periods)

Problem 2

(a) Define SMCE

- Prices, allocations, and *policy* $\{\tau_c, \tau_k, \tau_n, T_t\}$
- HH budget constraint
  - Consumption tax *adds* to expenditure
  - Labor, capital taxes *subtract* from income
- Firm maximizes profit
- Government *balances budget*
  - Issues no debt B_t
  - Makes no expenditures g_t
- Four markets clear
  - Consumption good: $c_t + x_t = y_t$
  - Asset market also clears

Problem 2

$$u(c_t, 1-n_t) = \frac{\left(c_t^{\mu}(1-n_t)^{1-\mu}\right)^{1-\sigma}}{1-\sigma}$$

$$\begin{aligned} u_c(c_t, 1-n_t) &= \mu\left[c_t^{\mu}(1-n_t)^{1-\mu}\right]^{-\sigma} c_t^{\mu-1}(1-n_t)^{1-\mu} \\ &= \frac{\mu\left[c_t^{\mu}(1-n_t)^{1-\mu}\right]^{1-\sigma}}{c_t} \\ u_n(c_t, 1-n_t) &= -(1-\mu)\left[c_t^{\mu}(1-n_t)^{1-\mu}\right]^{-\sigma} c_t^{\mu}(1-n_t)^{-\mu} \\ &= \frac{-(1-\mu)\left[c_t^{\mu}(1-n_t)^{1-\mu}\right]^{1-\sigma}}{1-n_t} \end{aligned}$$

Problem 2

(b) Write i_{t+1} in terms of taxes and allocations

- FOC wrt a_{t+1}:

$$1 + i_{t+1} = \frac{\lambda_t}{\lambda_{t+1}}$$

- **Option #1:** Replace λ_t w/ FOC wrt c_t, λ_{t+1} w/ FOC wrt c_{t+1}
- **Option #2:** Replace $\frac{\lambda_t}{\lambda_{t+1}}$ using FOC wrt k_{t+1}

Problem 2

(c) Write 3 equations characterizing **steady state**

- Impose $c_t = c_{t+1}$, $k_t = k_{t+1}$, so on!
- Euler equation uses FOCs wrt k_{t+1}, c_t, c_{t+1}
- Aggregate feasibility $\neq$ HH budget constraint
- MRS uses FOCs wrt n_t, c_t

(d) Use Euler from (c) to solve $\frac{k}{n}$ in terms of parameters, taxes **only**

(e) Use equation from (b) to solve i^*

- **Option #1:** Impose $c_t = c_{t+1}$, $n_t = n_{t+1}$
- **Option #2:** Plug in $\frac{k}{n}$ from (d)

Problem 1

Growth Rates and Tax Policy

(a) Define Arrow-Debreu competitive equilibrium.
  - ⟶ Be careful with policy variables (τ_c, τ_k, g)

(b) Derive Euler equation of household
  - ⟶ FOCs with respect to choice variables

(c) Find long-run growth rate of the economy: $\frac{c_{t+1}}{c_t}$

(d) Can government increase growth rate by subsidizing capital? $(\tau_k < 0)$

(e) Find optimal tax subsidy that maximizes representative household's welfare.
  - ⟶ Can be shown mathematically, but trust your intuition, too.

Problem 2

(a) Write down aggregate feasibility (aggregate or per person variables $k_t \equiv \frac{K_t}{\bar{N}}$)

(b) Write down profit maximization problem. Derive formulas for rental rate of capital r_t and wage w_t

(c) Define SMCE

(d) Write 3 equations in per-person variables that characterize equilibrium allocations

- Euler equation
- Aggregate feasibility
- Marginal rate of substitution consumption and leisure

(e) Find steady state capital-labor ratio $\frac{K}{H}$.

(f) How does labor income tax τ affect factor prices, capital-to-output ratio, labor supply, and total output?

Problem 3

AKH growth model with human capital accumulation

(a) Define TDCE

(b) Assume $\delta_k = \delta_h$. Find $\frac{c_{t+1}}{c_t}$ in terms of parameters and taxes.

(c) Tax reform – human capital investment x_t^h now exempt. Define TDCE.

- $\rightarrowtail$ Be careful editing government budget and household budget constraint.

(d) Find growth rates again.

(e) Holding tax rates fixed, which economy grows faster?

Problem 4

- Definitely the hardest problem on the problem set
- So far, we've treated population growth as an exogenous parameter. This model makes population growth *endogenous*.
- References missing on the problem set (actually useful for solving)
  - Barro, Robert J. and Gary S. Becker. (1989) "Fertility choice in a model of economic growth." *Econometrica* 57(2): 481–501.
  - Becker, Gary S. and Robert J. Barro. (1988) "A reformulation of the economic theory of fertility." *Quarterly Journal of Economics* 103(1): 1–25.

Problem 4

(a) Write down aggregate feasibility
- $\rightarrowtail$ R inclusive of depreciation, so law of motion of capital has $\delta = 0$
- $\rightarrowtail$ Raising children has two forms of cost:
  - Consumption good
  - Time-spent away from work

(b) Write down planning problem of "head of family" that takes initial population N_0, capital stock K_0 as given

(c) Use planner's FOCs to solve for *per capita* consumption c_t and population growth n_t. How do they depend on N_0, K_0?

Problem 4

(d) Write planner's problem recursively. Denote value function $V(K, N)$. Show that it is a contraction mapping.

- Blackwell's sufficient conditions

(e) Show that $V(K, N) = N^{1+\eta} v(k)$. Write Bellman equation that $v(k)$ solves.

- $k \equiv \frac{K}{N}$
- Plug in "guess" of $V(K, N)$ into Bellman and rearrange terms

(f) Show k' is constant and independent of k

- It's sufficient to write k' in terms of c, n, and parameters
- You already solved for c, n

Computation Exercise

Matlab installation instructions on eLC

(a) De-trend planner's problem (population, labor productivity grow)

(b) Write stationary planner's problem recursively. Determine k^{ss}:
  - ⟼ Write Euler equation
  - ⟼ Impose steady state condition: $V'(k) = V'(k')$

(c) Calibrate parameters $(A, \beta, \alpha, \delta)$

(d) Solve model **numerically by VFI**
  - ⟼ Lecture recording, slides, and references in eLC

(e) Use policy functions to simulate capital, consumption, and output