ECON 8040 – TA12

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Today's Session

- PS7 due **Friday, November 10**
- PS8 due **Friday, November 17**
- Computation Exercise due **Thursday, November 30**
- PS9 due **Friday, December 1** at 9:00 a.m.
- Final exam **Thursday, December 7, 3:30–6:30 p.m.**

Problem 1

(a) Write down planner's problem
- ⟶ Carefully write down all choice variables
- ⟶ Business sector aggregate feasibility: output of the business sector can be consumption of business sector good, invested in business capital, or invested in home capital
- ⟶ Home-produced consumption good cannot be invested in either form of capital

(b) Write the Bellman equation
- ⟶ Carefully write down state and control variables
- ⟶ Replace constraints in objective where convenient
- ⟶ Don't forget to write down all remaining constraints!

Problem 1

(c) Write down FOCs and envelope conditions

- ⟼ If derivatives are too difficult, you may want to adjust the Bellman you wrote down in (b)

(d) Write equations that characterize the steady state

- ⟼ Feasibility and first-order conditions in steady state
- ⟼ There should be as many equations as "unknowns." In principle, the system can be solved.

Problem 2

Two nations vary in discount factor β_i.

(a) Write steady state capital in case of autarky (i.e., no trade). Which country has higher steady-state capital? Does the result make sense?

(b) Write planner problem when capital can cross borders.

- $\rightarrowtail$ One feasibility constraint
- $\rightarrowtail$ Find steady state $\frac{c_t^2}{c_t^1}$ and steady state capital in each country.

Problem 3

Given Cobb-Douglas production function, law of capital motion, and three types of preferences $u(c, 1-n)$

log-constant Frisch elasticity	$\log c - \psi \frac{n^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$
log-log	$\phi \log c + (1-\phi)\log(1-n)$
Cobb-Douglas	$\frac{\left(c^{\phi}(1-n)^{1-\phi}\right)^{1-\sigma}}{1-\sigma}$

For each:

(a) Write down recursive planning problem and FOCs, envelope conditions

(b) Characterize steady state allocations in terms of k and n

Problem 4

Economy with distinct consumption and investment goods.

(a) Write planner's problem and planner's FOCs

$\rightarrowtail$ There are two aggregate feasibility constraints on planner. It may be helpful to think of what market clearing conditions would be in a competitive equilibrium.

(b) Write planner's problem recursively and take FOCs, envelope conditions.

$\rightarrowtail$ Carefully write down state and control variables.

Problem 5

Model of endogoneous mortality from Hall and Jones (2007).

(a) Write down planner's problem
   - $\rightarrowtail$ Assume discount factor $\beta = 1$
   - $\rightarrowtail$ Be careful with choice variables, feasibility and law of motion

(b) Write planner's problem recursively. Does it satisfy Blackwell's sufficient conditions?
   - $\rightarrowtail$ What is the state variable? i.e., what does planner need to know at beginning of period to make optimal allocations?
   - $\rightarrowtail$ Write down feasibility and law of motion constraints

Problem 5

(c) Show that value function of form vN solves the Bellman. What is v?

(d) Write down FOC. What is p?

(e) Use given $u(c)$ and $f(h)$ to find health expenditure share of income, i.e., $s \equiv \frac{ph}{y}$

(f) Comparative statics: how does s change in parameters, p?

Problem 1

The Lucas Tree. (Maybe) a helpful analogy: Imagine a shipwrecked crew on an island. The only consumption good available to crew is fruit of a tree on the island. Each period t tree produces fruit d_t. Crew member with s_t shares of tree is entitled to $s_t d_t$ fruit, and shares are traded at price p_t^s. Crew can also trade one-period risk-free bonds b_t at price p_t^b.
The budget constraint is

$$c_t + p_t^b b_{t+1} + p_t^s(s_{t+1} - s_t) = b_t + s_t d_t$$

(a) Define SMCE. State *all* allocations and market clearing conditions. Write utility as $u(c)$.

Problem 1

(b) Write Recursive Competitive Equilibrium.

- $\rightarrowtail$ Rewrite budget constraint and use hint that $w \equiv b + s(p^s(d) + d)$
- $\rightarrowtail$ Allocations in (a) are *policy functions*
- $\rightarrowtail$ Prices in (a) are functions of dividends
- $\rightarrowtail$ State variables: (w, d)
- $\rightarrowtail$ Control variables: (c, s', b')
- $\rightarrowtail$ Refer to example in lecture notes

You don't need to do part (b) to solve parts (c) and (d)

Problem 1

Use SMCE definition in (a) to solve stock price p_t^s and bond price p_t^b.

(c) Assume $d_t = 1$. No utility form provided, just keep utility as $u(c)$

(d) Assume $u(c) = \log c$ and $d_t = \begin{cases} 1 & t = 0, 2, 4, \ldots \\ 2 & t = 1, 3, 5, \ldots \end{cases}$

Problem 2

Sequential markets economy w/ government

(a) Define SMCE

- ⟼ Prices, allocations, and *policy*
- ⟼ HH budget constraint includes taxes and transfers
- ⟼ Firm maximizes profit
- ⟼ Government tax revenues equal lump-sum transfers
- ⟼ Four markets clear

Problem 2

(b) Write i_{t+1} in terms of taxes, allocations
   - $\rightarrowtail$ FOC wrt a_{t+1}

(c) Write 3 equations determining **steady state** allocations
   - $\rightarrowtail$ Euler equation
   - $\rightarrowtail$ Aggregate feasibility
   - $\rightarrowtail$ MRS consumption / leisure

Problem 2

(d) Write capital-labor ratio as function of taxes, parameters

$\longmapsto$ Use **Euler equation**

(e) Find steady state interest rate

$\longmapsto$ Use equation from (b)

Computation Exercise

Matlab installation instructions on eLC

(a) De-trend planner's problem (population, TFP grow)

(b) Write stationary planner's problem recursively. Determine k^{ss}:

- $\rightarrowtail$ Write Euler equation
- $\rightarrowtail$ Impose steady state condition: $V'(k) = V'(k')$

(c) Calibrate parameters $(A, \beta, \alpha, \delta)$

(d) Solve model **numerically by VFI**

- $\rightarrowtail$ Lecture recording, slides, and references in eLC

(e) Use policy functions to simulate capital, consumption, and output