ECON 8040 – TA10

Michael Kotrous

University of Georgia

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Today's Session

- PS6 due **Friday, October 27**
- Midterm Exam #2, Tuesday, October 31

Problem 1

Static model of indivisible labor supply (i.e., $h = 0$ or $h = 1$)

a) Define competitive equilibrium
   - $\rightarrowtail$ Use i superscript for HH allocations (no representative HH)
   - $\rightarrowtail$ HH makes allocation k^{i*}. Optimal choice is $k^{i*} = \bar{k}^i$
   - $\rightarrowtail$ HH chooses h^{i*}, not ℓ^{i*}
   - $\rightarrowtail$ WLOG normalize $p^* = 1$
   - $\rightarrowtail$ Firm has CRS technology
     - Representative firm
     - Zero profit in equilibrium

b) Assume $u(c, h) = \log(c) - h\log(7)$, $\alpha = 1/3$. Solve aggregate allocations, prices.
   - $\rightarrowtail$ Don't write FOC wrt h
   - $\rightarrowtail$ Households who work (i.e. $h = 1$) and don't work ($h = 0$) have equal utility

Problem 2

Static model of expenditure shares agriculture, manufacturing, goods

a) Find expenditure shares for each good, i.e find $\frac{p_i c_i}{y}$

$$\max_{\{c_a, c_m, c_s\}} \phi_a \log(c_a - \bar{c}_a) + \phi_m \log(c_m) + \phi_s \log(c_s + \bar{c}_s)$$

subject to $p_a c_a + p_m c_m + p_a c_a = y; \lambda$
Sketch:

- ⟼ Divide FOC wrt c_a by FOC wrt c_m
- ⟼ Divide FOC wrt c_s by FOC wrt c_s
- ⟼ Write $\frac{p_a c_a}{y}$, $\frac{p_s c_s}{y}$ in terms of parameters, $\frac{p_m c_m}{y}$
- ⟼ Divide budget constraint by y; replace $\frac{p_a c_a}{y}$, $\frac{p_s c_s}{y}$
- ⟼ Solve for $\frac{p_m c_m}{y}$ in terms of parameters
- ⟼ Replace $\frac{p_m c_m}{y}$ in equations in step 3 to finish

Problem 4

Two-period sequential market economy

The correct endowments are:

$$(e_0^1, e_1^1) = (1, 0)$$
$$(e_0^2, e_1^2) = (0, 1)$$

See Fall 2021 Midterm Solutions document for answer

Problem 1

Ag, manufacturing, services share model

a) Set up similar to PS4, Problem 2
b) Pull data for ag, manufacturing, and services consumption in U.S.
c) "Calibrate" the parameters of the model
d) Interpret $\bar{c}_a$ and $\bar{c}_s$ parameters
e) Compare expenditure shares path predicted by model against actual data

Problem 2

Define competitive equilibrium for static model of continuous labor supply

- ⋆ State all equilibrium allocations, prices
  - ⟼ WLOG normalize price of consumption good $p^* = 1$
- ⋆ Define household problem
  - ⟼ Representative household
  - ⟼ Labor and leisure must add up to time endowment
- ⋆ Define firm problem
  - ⟼ What are firm profits in equilibrium?
- ⋆ Three markets clear

Problem 3

Solve static model of continuous labor supply, assuming

$$u(c,\ell) = \frac{1}{1-\sigma}\left[\left(c^{\phi}\ell^{1-\phi}\right)^{1-\sigma} - 1\right] \qquad F(K,N) = AK^{\alpha}N^{1-\alpha}$$

- ⋆ Assume $\phi \in (0,1)$, $\sigma \geq 1$
- ⋆ You may assume $\bar{k} = 1$ (but you don't have to)
- ⋆ What is k^*?
- ⋆ Rewrite HH problem in terms of two choices: c, ℓ
- ⋆ Write MRS of c, ℓ
- ⋆ Use budget constraint, firm FOCs to write equation with one unknown: ℓ^*
- ⋆ Use ℓ^* to solve other allocations, prices

Problem 4

Solve static model of continuous labor supply, assuming

$$u(c,\ell) = c - \frac{1}{1+\frac{1}{\varepsilon}}(1-\ell)^{1+\frac{1}{\varepsilon}} \qquad F(K,N) = AK^{\alpha}N^{1-\alpha}$$

- ⋆ Assume $\varepsilon > 0$
- ⋆ Write HH problem with two choices: c, h (or c, ℓ)
- ⋆ Use FOCs to write w in terms of h
- ⋆ Use firm's FOC to write equation with one unknown: h^*
- ⋆ Use h^* to find other allocations, prices

Problem 1

a) Write the Bellman equation

b) Solve the Bellman using the provided guess and following the lecture notes

   $\rightarrowtail$ Solve policy function k' and value function $v(k)$

c) Use the policy function for k' to find $\frac{k_{t+1}}{k_t}$ and $\frac{c_{t+1}}{c_t}$

Problem 2

a) Write down FOC for n using $F(k, k')$
Write Bellman equation using $F(k, k')$

b) Assume full depreciation ($\delta = 1$)
Find FOC for k' using provided guess: $V(k) = A + B\log(k)$

c) Write n in terms of parameters and B

d) Replace k' and n in guess of $V(k)$ to solve B

e) Solve for policy functions n, k', and c as function of state k

Problem 3

a) Write planning problem $w(k_0, h_0)$ (k_0, h_0 is given initial capital stock)

b) Write the planning problem recursively

c) Assume full depreciation ($\delta = 1$) and use guess-and-verify to solve:
   - $\rightarrowtail V(k, h)$
   - $\rightarrowtail k'(k, h)$
   - $\rightarrowtail h'(k, h)$

Problem 4

a) Rewrite the problem so $\{k_{t+1}\}_{t=0}^{\infty}$ is only choice variable

b) Write the problem recursively using two equations
   - $\rightarrowtail$ $v(k, \theta_L)$
   - $\rightarrowtail$ $v(k, \theta_H)$
   - $\rightarrowtail$ You know how state θ_t evolves

c) Solve the Bellman equations using guess-and-verify

d) Find policy functions $g(k, \theta_L)$ and $g(k, \theta_H)$